14.1 INTRODUCTION

In many applications it is necessary to maintain the speed of electric motors constant. In general, speed tends to fall as the motor is loaded; on other hand, for a number of electric motors, it is possible to raise the speed by increasing the applied voltage. Phase controlled rectifiers are widely used in the speed control of d.c. motors, the control circuits are designed in such a way that, as loading increasing, $\alpha$ is made smaller thus raising the applied voltage, and hence attempting to maintain the speed constant.

DC-Drives are widely used in applications requiring adjustable speed and good regulation; Control of the speed of a d.c. motor is extensively used below and above the base (or rated) speed. The methods of control are simple and less expensive than those applicable to a.c. motors. The converters used are controlled rectifiers or choppers. If the supply is a.c., controlled rectifiers can be used to convert a fixed a.c. voltage into a variable voltage d.c. supply. Several types of controlled rectifier circuits are available: single-phase and three-phase half-wave and full-wave. In general, the type of controlled rectifier adopted depends on the motor power:

- Up to 7.5 kW (10 hp) a single-phase a.c. bridge is customary unless the supply impedance and other users lead to unacceptable supply disturbance, where a three-phase control would be used.
- For more than 7.5 kW, the three-phase bridge converter is universally used with thyristors and diodes where non-regenerative braking is acceptable.
14.2 CLOSED-LOOP VARIABLE SPEED D.C. DRIVE

Regardless of the type of the power circuit (single or three-phase rectifier or chopper) or its mode of operation (i.e. continuous or discontinuous current mode) the basic requirements for closed-loop variable d.c. drive is essentially the same.

Firing of an SCR switch is accomplished by a train of pulses applied to the gate. These pulses are synchronized with the a.c. supply (in case of rectifier circuits), the amplitude of the pulse is unimportant as long as it drives sufficient gate current (in case of thyristors, GTO or power transistors). The important factor is the firing angle $\alpha$, since by varying $\alpha$ the applied voltage $V_a$ to the motor and the speed of the driven motor are controlled. Typical closed-loop variable speed drive is shown in Fig.14.1.

![Fig.14.1 Typical closed-loop variable speed d.c. drive.](image)

14.2.1 The Triggering (Firing) Circuit

The train of pulses is generated by triggering circuit. Several type of triggering circuits is used in practice. However, in closed-loop d.c. drive the study will be limited here to their function, which can be defined as in Fig.14.2 : The firing circuit generates a train of pulses whose delay angle $\alpha$ is a function of the input signal $Y$, which may be a voltage or current input to the firing angle control circuits shown in Fig 14.2.

![Fig.14.2 Function of the triggering circuit.](image)
14.2.2 Control Signal and Components

The input signals to the triggering circuit, Y, determine the firing angle $\alpha$ and hence the speed of the driven motor. Y itself is determined, essentially by two types of signal:

1) A reference signal, this is a voltage derived from the a.c. supply or from d.c. supply in case of choppers. It is pre-set (manually, say, by a potentiometer) at some value, and remains constant throughout the entire operation.

2) Feedback signal (or signals), these are signals proportion to motor speed, voltage or current. Cleary, the feedback signals vary as the motor is loaded.

The signal flow and the function of various control components may be understood by referring to Fig.14.3 which shows a complete drive composes of the following basic parts:

Fig.14.3 SCR control system driving a separately-excited d.c. motor ($J$ and $B$ are, inertia and friction of the motor and load respectively).

(a) Power section: A converter (rectifier or chopper with or without freewheeling diode).

(b) motor and load ; in this case the motor is a separately-excited d.c. motor whose main field flux is constant, the load is assumed to be only inertial (friction is neglected).

(c) Control section: This section comprises the following items,
   (i) Triggering (firing) circuit.
   (ii) Means for sensing $n$ (o), voltage $V_{av}$ and current $I_a$.
   (iii) Reference comparison, and signal processing circuits the blocks marked $F_y$, $F_i$, $F_v$, and $F_\omega$ may be an amplifier, attenuator, or other signal processors.
14.3 SPEED CONTROL

DC Motor Speed Control Basic Theory

Before considering the details of speed control, it is required to define the following related terms:

*Pre-set speed*: The speed to which the drive is pre-set, for example by means of adjusting the reference voltage in the control circuit \( (n_p \text{ in Fig.14.4}) \). Maximum setting of \( n_p \) is called the base speed \( (n_{p_{\text{max}} \text{ in Fig.14.4}}) \). Minimum possible setting of \( n_p \) is \( n_{p_{\text{min}}} \) in the figure.

![Graph of Speed and Torque Characteristics](image)

**Fig.14.4 Speed-torque characteristics of adjustable-speed drives.**

*Speed regulation*: The change in speed from no-load to full-load in percent of the pre-set speed. Thus at a pre-set speed \( n \) in Fig.14.4:

\[
\text{speed regulation} = \frac{\Delta n_{fl}}{n_p} \times 100\% \quad (14.1)
\]

Sometimes the speed regulation is given in percent of the base speed \( n_{p_{\text{max}}} \), i.e.

\[
\text{speed regulation} = \frac{\Delta n_{fl}}{n_{p_{\text{max}}}} \times 100\% \quad (14.2)
\]

In controlled drives, speed regulation ranges from 1% to 5%.

*Speed range*: Is the ratio of the base speed \( n_{p_{\text{max}}} \) to the minimum possible speed setting \( n_{p_{\text{min}}} \).
By defining the base speed range, \( n_{pmin} \) is defined, and hence the entire range of speeds over which the motor can be controlled. A typical speed range is 6:1. In general, \( n_{pmin} \) is determined by limits of temperature rise in the motor.

Having defined the above technical terms, we can state that the objective of speed control is to operate the drive with small speed regulation over a wide speed range.

14.3.1 Open-Loop Operation

If, in Fig.14.3, the speed, voltage and current feedback signals are removed, then \( Y \), and hence the firing angle \( \alpha \) are determined solely by the reference signal. For a given pre-set speed, the reference signal is constant; thus the speed drops according to the internal characteristics of the motor, and speed regulation is very large.

14.3.2 Closed-Loop Operation

Speed regulation is greatly improved if a feedback signal proportional to speed is used, the control system compares this feedback signal with the reference signal; the difference, \( Y' \) in Fig.14.3, is amplified to control the firing or triggering circuit in such a way as to maintain the speed constant. At no-load, the feedback signal subtracts from the reference voltage to yield a certain \( Y' \) (corresponding to the pre-set speed \( n_p \)); as the motor is loaded, the speed tends to fall and the feedback signal decreases so that \( Y' \) increases; thus \( \alpha \) is decreased to increase the motor voltage, tending to restore the speed to its original pre-set value; in fact the speed does not reach the original value, but slightly less. This difference, or error is defined by,

\[
\Delta n = n_p - n
\] 

(14.3)

which produces the necessary change in \( \alpha \). Thus the speed regulation ability of the control section depends on two main factors:

1. How well the feedback signal depicts motor speed; this depends on the quality of the speed sensing means and the components present in the feedback path.
2. How much error in speed, \( \Delta n \) is required to produce a given change in the firing angle \( \alpha \). It is desirable that only a small \( \Delta n \) is sufficient to produce large changes in \( \alpha \). Clearly, this factor is related to the amount of amplification available in the control circuit (for example in the block \( F_Y \) of Fig.14.3).
**Speed sensing**

In section 14.2 it was clear that we need a feedback signal which represents the speed in precise manner. The most obvious method of speed sensing is by means of a tachogenerator mounted on the shaft of the motor under control. The tachogenerator generates a voltage proportional to the motor speed with high degree of accuracy (linearity), which is its main advantage; however, the installation of a tachogenerator constitutes a costly item in the drive. A cheaper method of speed sensing is based on motor basic equation which is

\[ V_{a(\text{av})} = I_{a(\text{av})} R_a + K_e \Phi n \]  \hspace{1cm} (14.4)

If the main field flux \(\Phi\) is constant, and the resistive drop \(I_{a(\text{av})} R_a\) is negligible, then the average armature voltage \(V_{a(\text{av})}\) is proportional to speed \(n\). Thus the armature voltage may be used as the feedback signal represents the speed. This method is cheap because it uses little additional hardware, but it is inaccurate because the feedback signal used is not very good representation of motor speed due to the above assumption. However, this last method may be improved considerably by the use of current feedback. Let us re-write Eq.(14.4) in the form

\[ V_{av} - I_{av} R_a = K_e \Phi n \]  \hspace{1cm} (14.5)

If \(\Phi\) is constant, then the term on the left hand side is in fact, proportional to speed. Thus a feedback signal composed of two components is used: the first component is proportional to armature voltage as before, but the second component is proportional to armature current and is subtracted from the first component (in effect, it is added to the reference signal) to yield an overall feedback signal proportional to the speed as desired. It may be said that current feedback used in this way compensates for the resistive drop of the armature circuit. The current multiplier representing \(R_a\) is usually set by test for optimum performance. This method is effective and quite cheap, but it may suffer from stability problems due to positive current feedback.

**Amplification**

As mentioned in subsection 14.2.2, the amplification, or gain, available in the control loop affects the speed regulation ability of the drive. As the gain increases, a small error in speed \((\Delta n = n_p - n)\) is sufficient to produce large changes in the firing angle, these changes being, in turn , sufficient to keep \(\Delta n\) small. Usually, the amplifier stage is located after the comparator stage. With small gains, a relatively large error in speed is needed to advance \(\alpha\) sufficiently; in effect speed regulation is large.
Very close speed regulation is achieved by the use of tachogenerator speed sensing together with an operational amplifier; however, the system is expensive, and is used only where the application justifies its high cost. Moreover, high gains may give rise to stability problems, thus limiting the range of operation of the motor.

**Transient effects**

So far, our analysis has been limited to steady-state conditions where the objectives are to obtain a small value of speed regulation over a wide speed range. Inevitably, transient effects arise whenever the drive experience sharp changes in loading, voltage, or speed. Ideally, after any disturbance, the system should settle quickly to the new steady-state. Any oscillations that arise being damped with a short time constant.

In practice, closed-loop control may give rise to instability over certain ranges of operation, where transient may cause undamped oscillations. It is interesting to note that the devices which improve steady-state performance tend to be the ones that cause transient problem. In particular, transient performance may suffer from high gains in the control loop, and from the use of positive current feedback, especially at high loads. In these cases we shall be interested not only in steady state performance (speed regulation and speed range), but also in the transient effects of such disturbances as starting and run-up, sudden loss of speed, and sudden load shedding. It will also be useful to remember that instability, unless very severe, occurs over only certain ranges of speed and loading.

### 14.4 DC MOTOR CONTROL CHARACTERISTICS

To study the d.c. motor control characteristics from the control theory point of view, let us consider a separately-excited d.c. motor at loading condition controlled by a power electronics converter as depicted in Fig.14.5. The motor speed is adjusted by setting reference or control voltage, $v_r$.

Assuming the converter of gain $K_c$ has a linear power characteristics, then, the output voltage of the converter is

$$v_a = K_c v_r$$

is directly supplied to the armature circuit. Therefore, the closed-loop transfer function of the motor could be evaluated easily by considering first its open-loop transfer function.
14.4.1 Open-Loop Transfer Function of d.c. Motor

To find the open-loop transfer function of a d.c. motor, consider the following basic equations of the d.c. motor:

The general motor electrical equation referred to Fig.14.5 (armature circuit),

\[ v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \]  \hspace{1cm} (14.7)

The induced back \textit{emf}

\[ e_a = K \phi \omega \]  \hspace{1cm} (14.8)

Substituting Eq. (14.8) in Eq. (14.7) yields

\[ v_a = R_a i_a + L_a \frac{di_a}{dt} + K \phi \omega \]  \hspace{1cm} (14.9)

Also, the mechanical equations of the motor are,

\[ T = T_L + J \frac{d\omega}{dt} + B \omega \]  \hspace{1cm} (14.10)

and the induced (developed) torque

\[ T = K\phi i_a \]  \hspace{1cm} (14.11)
The Laplace transform of the above four equations (14.8), (14.9), (14.10) and (14.11), assuming zero initial conditions are,

\[ v_a(s) = R_a I_a(s) + L_a s I_a(s) + K\phi \omega(s) \]  \hspace{1cm} (14.12)

\[ E_a(s) = K\phi \omega(s) \]  \hspace{1cm} (14.13)

\[ T(s) = T_L(s) + Js\omega(s) + B\omega(s) \]  \hspace{1cm} (14.14)

\[ T(s) = K\phi I_a(s) \]  \hspace{1cm} (14.15)

From equations (14.12), and (14.13) the current is given by

\[ I_a(s) = \frac{V_a(s) - E_a(s)}{R_a(1 + s\tau_a)} = \frac{V_a(s) - K\phi \omega(s)}{R_a(1 + s\tau_a)} \]  \hspace{1cm} (14.16)

where \( \tau_a = \frac{L_a}{R_a} \) = electrical time constant of the motor.

From equations (14.14), and (14.15)

\[ \omega(s) = \frac{T(s) - T_L(s)}{B(1 + sJ/B)} = \frac{K\phi I_a(s) - T_L(s)}{B(1 + s\tau_m)} \]  \hspace{1cm} (14.17)

where \( \tau_m = \frac{J}{B} \) = mechanical time constant of the motor.

The open-loop block diagram of the motor represented by Eqs.(14.13), (14.16) and (14.17) is shown in Fig.14.6. Two possible disturbances may occur to the motor, the terminal voltage \( V_a \) (or reference voltage \( V_r \) ), and the load torque \( T_L \). The steady-state responses can be determined by combining the individual response due to \( V_a \) and \( T_L \) as follows:

![Open-loop block diagram representation of a separately-excited d.c. motor.](image-url)
The response to change in \( V_a \) (or \( V_r \)) is obtained by setting \( T_L = 0 \) (no load condition), hence Fig.14.6 will represent the general negative feedback system with transfer function:

\[
TF = \frac{G(s)}{1 + G(s)H(s)}
\]  

(14.18)

\[
\frac{\omega(s)}{V_a(s)} = \frac{\frac{K\Phi}{R_a(1+s\tau_a)B(1+s\tau_m)}}{1 + \frac{K\Phi}{R_a(1+s\tau_a)B(1+s\tau_m)}.K\Phi}
\]

\[
= \frac{K\Phi}{R_a B(1+s\tau_a)(1+s\tau_m) + (K\Phi)^2}
\]

\[
= \frac{K\Phi/R_a B}{s^2\tau_a \tau_m + s(\tau_a + \tau_m) + 1 + (K\Phi)^2/R_a B}
\]  

(14.19)

It is clear from Eq.(14.19) that the motor forms a second order system with inherent feedback.

From Fig.14.5, and the relation \( v_a = K_c v_r \), the response to step change in the control voltage \( V_r(s) = V_a(s)/K_c \) can be obtained as,

\[
\frac{\omega(s)}{V_r(s)} = \frac{K_c K\Phi/R_a B}{s^2\tau_a \tau_m + s(\tau_a + \tau_m) + 1 + (K\Phi)^2/R_a B}
\]  

(14.20)

The response to change in the torque \( T_L \) is obtained by setting \( V_a = 0 \) (or \( V_r = 0 \)), hence the block diagram for step change in load torque will be as depicted in Fig.14.7.

For positive feedback, the transfer function of the block diagram of Fig.14.7 is,
The steady-state relationship of a change in speed $\Delta \omega$ due to step change in the reference voltage $\Delta V_r$ and a step change in load torque $\Delta T_L$, can be found from Eqs.(14.20) and (14.21) respectively by applying the final value theorem in which we substitute $s = 0$ in both equations to give,

$$\Delta \omega = \frac{K_c K\Phi}{R_a B + (K\Phi)^2} \Delta V_r$$

(14.23)

$$\Delta \omega = \frac{-R_a}{R_a B + (K\Phi)^2} \Delta T_L$$

(14.24)

For special case, if the viscous friction $B = 0$, the above equations become,

$$\Delta \omega = \frac{K_c \Delta V_r}{K\Phi} = \Delta \omega_o \quad \text{no - load speed change}$$

(14.25)

$$\Delta \omega = \frac{-R_a}{(K\Phi)^2} \Delta T_L \quad \text{change in } \omega \text{ due to loading}$$

(14.26)

or \( \omega = \Delta \omega_o + \Delta \omega = \frac{K_c \Delta V_r}{K\Phi} - \frac{R_a}{(K\Phi)^2} \Delta T_L \)

Notes:

(1) The above transfer functions are valid for continuous current operation only.

(2) The motor is always stable in open-loop operation.
14.4.2 Closed-Loop Transfer Function

Open-loop control system for separately-excited d.c. motor of Fig.14.1 may be changed to closed-loop control system by attaching speed sensor to the motor shaft. The output of the sensor (could be a tachogenerator or speed encoder) which is proportional to the speed, is amplified by a factor \( K_t \) and compared with a reference voltage \( v_r \) to give an error signal \( v_e \). The block diagram for a typical closed-loop control system of a separately-excited d.c. motor is shown in Fig.14.8.

![Block diagram for a typical closed-loop control system of a separately-excited d.c. motor.](image)

The closed-loop step response due to a change in reference voltage can be obtained from Fig.14.8 with \( T_L = 0 \). The transfer function is,

\[
\frac{\omega(s)}{V_r(s)} = \frac{K_c K \phi / R_a B}{s^2 \tau_a \tau_m + s(\tau_a + \tau_m) + 1 + [(K \phi)^2 + K_t K_c K \phi] / R_a B} \tag{14.27}
\]

The closed-loop step response due to a change in load torque \( T_L \) can be obtained from Fig.14.8 by setting \( V_r = 0 \), hence the transfer function is found to be,

\[
\frac{\omega(s)}{T_L(s)} = \frac{1}{B(1 + s \tau_m)} = \frac{-1}{B(1 + s \tau_a)} \tag{14.28}
\]

The steady-state relationship of a change in speed \( \Delta \omega \) due to step change in the reference voltage \( \Delta V_r \) and a step change in load torque \( \Delta T_L \), can be found from equations (14.27) and (14.28) respectively by applying the final value theorem in which we substitute \( s = 0 \) in both equations to give,
Example 14.1

A 45 kW, 240 V, 2000 rpm, separately-excited d.c. motor is to be speed controlled by a thyristor converter as shown in Fig.14.5. The machine constant $K\Phi = 1.0$ V.s / rad. The armature resistance $R_a = 0.1 \Omega$ and the viscous friction constant $B = 0.1$ Nm.s/rad. The speed sensor amplification factor $K_t = 100$ mV.s /rad and the gain of the power control $K_c = 100$. It is required to determine the following:

(a) The rated torque of the motor.
(b) The reference voltage $V_r$ to drive the motor at rated speed.
(c) The speed at which the motor develops the rated torque if the reference voltage is kept constant.
(d) The motor speed if the load torque is increased by 15% of the rated value.
(e) The motor speed if the reference voltage is reduced by 15%.

Solution

(a) The rated speed $\omega_R$ is

$$\omega_R = \frac{2000 \times 2\pi}{60} = 209.44 \text{ rad/s.}$$

Rated torque $T_R$ is

$$T_R = \frac{P}{\omega_R} = \frac{45000}{209.44} = 214.85 \text{ Nm}$$

(b) Since for open-loop $V_a = K_c \cdot V_r$, Eq.(14.23) gives

$$\omega = \frac{K_c K\Phi}{R_a B + (K\Phi)^2} V_r = \frac{K_c V_r K\Phi}{R_a B + (K\Phi)^2} = \frac{V_a K\Phi}{R_a B + (K\Phi)^2}$$
\[ V_a = \frac{\omega [R_a B + (K \phi)^2]}{K \phi} = \frac{209.44 \left[ (0.1 \times 0.1) + (1.0)^2 \right]}{1.0} = 211.53 \text{ V} \]

and the feedback voltage, \( E_a' = K_r \omega = 100 \times 10^{-3} \times 209.44 = 20.944 \text{ V} \).

With closed-loop control, the error voltage \( V_e = V_r - E_a' \),

\[ V_a = V_c K_c = (V_r - E_a') K_c \]

From which,

\[ V_r = \frac{V_a + K_c E_a'}{K_c} = \frac{211.53 + 20.944 \times 100}{100} = 23.05 \text{ V} \]

(c) For \( V_r = 23.05 \text{ V} \) and \( T_R = 214.85 \text{ Nm} \), from Eq.(14.30)

\[ \Delta \omega = \frac{-0.1 \times 214.85}{0.1 \times 0.1 + (1.0)^2 + 100 \times 10^{-3} \times 100 \times 1.0} = -1.951 \text{ rad/s} \]

The speed at rated torque = 209.44 – 1.951 = 207.49 rad / s.

(d) If the load torque is increased by 15 % of the rated value:

\[ \Delta T_L = 1.15 \times 214.85 = 247.07 \text{ Nm} \]

\[ \Delta \omega = \frac{-0.1 \times 247.07}{0.1 \times 0.1 + (1.0)^2 + 100 \times 10^{-3} \times 100 \times 1.0} = -2.243 \text{ rad/s} \]

The speed at rated torque = 209.44 – 2.243 = 207.197 rad / s.

(e) Now, \( \Delta V_r = -0.15 \times 23.05 = -3.4575 \text{ V} \), From Eq.(14.29),

\[ \omega = \frac{100 \times 1.0}{0.1 \times 0.1 + (1.0)^2 + 100 \times 10^{-3} \times 100 \times 1.0} \times (-3.4575) \]

\[ \omega = -31.4 \text{ rad/s} \]

The speed at rated torque = 209.44 – 31.4 = 178.04 rad / s.

14.5 PRACTICAL CLOSED-LOOP CONTROL SYSTEM FOR D.C. MOTOR WITH SPEED AND CURRENT CONTROLLERS

In practice d.c. motor closed-loop control schemes do not contain speed feedback only. The scheme must also contain current feedback to meet the load torque which depends on the armature current. When the motor is operating at a particular speed and suddenly load is applied, the
speed will fall and the motor will take time to restore to its original speed. Therefore, a speed feedback with an inner current loop gives faster response to any change in speed, load torque or the supply voltage as depicted in Fig.14.9.

The function of the current loop is to react with the sudden change of the load torque under transient conditions of operation such as starting, breaking, speed reversal, and torque disturbances. The supply current which is the a.c. input current (equivalent to the armature current), sensed by the current transformer CT, rectified and filtered by an active filter to remove ripples to give the actual d.c. armature current $I_a$. The output of the speed controller is applied to a current limiter which sets the current reference $I_{ar}$ for the current loop to a safe value, no matter how high the speed error is. The current error ($I_a - I_{ar}$) is applied to the current controller to adjust the firing angle $\alpha$ of the thyristor unit (or $\gamma$ for a chopper) to bring the motor speed to the desired value by increasing or decreasing the armature voltage. The reference current is the output of the speed controller.

The function of the current limiter is to saturate and limit the current in case of any large increase in the value of the reference current $I_{ar}$ due to any positive speed error resulting from either increase in the speed command or an increase in the load torque. This variation in $I_{ar}$ is processed by the current controller whose output adjusts the firing angle of the converter to correct the speed error and finally settled to a new $I_{ar}$, which makes the motor torque equal to the load torque and the speed error closed to zero.

The current control loop is generally faster than the speed control loop, therefore it is preferred that the current loop is designed first by assuming
the motor operates at constant speed. Once the current loop is designed, the speed control loop is then considered to satisfy torque demand.

14.5.1 Speed and Current Controllers Transfer Functions

Types of the Controller-Review:
A controller for a converter-fed variable-speed drive is an electronic device whose input and output values are electrical signal. By suitable electrical design it is possible to obtain the required control action. The four principal types of controllers are:

(1) P-Controller (Proportional-action):
P-controller is mostly used in first order system to stabilize the unstable process. The main usage of the P-controller is to decrease the steady state error of the system. As the proportional gain factor $K$ increases, the steady state error of the system decreases. However, despite the reduction, P-control can never manage to eliminate the steady-state error of the system. As we increase the proportional gain, it provides smaller amplitude and phase margin, faster dynamics satisfying wider frequency band and larger sensitivity to the noise. Hence, this type of controller can only be used when the system is tolerable to a constant steady-state error. In addition, it can be easily concluded that applying P-controller decreases the rise time and after a certain value of reduction on the steady state error, increasing $K$ only leads to overshoot of the system response. P-control also causes oscillation if sufficiently aggressive in the presence of lags and/or dead time. The more lags (higher order), the more problem it creates.

Operational Amplifier P-Controller

Fig.14.10 shows a P-controller using operational-amplifier. The transfer function of the circuit is,

![Fig.14.10 Op-amplifier P-controller.](image)

\[ \frac{V_i}{R_1} = i_1 \quad \text{and} \quad \frac{V_0}{R_2} = -i_2 \]

Since $i_1 = i_2$

\[ \therefore \quad \frac{V_0}{V_i} = -\frac{R_2}{R_1} = K_c \quad \text{(Controller gain)} \quad (14.31) \]
Hence with p-controller the output is directly proportional to the amplitude value of the input, i.e. \( V_o = K_c V_i \).

**P- Speed controller**

In this type of controllers, the transfer function shows that an input change in \( V_r \) results in large sudden change in current that decays slowly, this transient overcurrent is undesirable from the stand point of converter rating and protection. This particularly the case for starting or other large change.

**P- Current controller**

The current drawn by the motor is fed back by the inner loop with current transducer. Relatively there is small overshoot and in practice, there will be a delay due to the armature circuit electrical time constant together with the converter delay. Sometimes a filter is required to reduce the ripple in the current sensor element or in the tachogenerator itself.

(2) PI-Controller (Proportional plus integral action):

PI-controller is mainly used to eliminate the steady state error resulting from P-controller. However, in terms of the speed of the response and overall stability of the system, it has a negative impact. This controller is mostly used in areas where speed of the system is not an issue. Since PI-controller has no ability to predict the future errors of the system it cannot decrease the rise time and eliminate the oscillations. Fig.14.11 shows a PI-controller using Op-amplifier. The transfer function of the circuit is obtained as,

\[
G_c(s) = \frac{v_o}{v_i} = -\frac{R_2}{R_1} \left(1 + \frac{1}{sC_2R_2}\right) = K_c \left(\frac{T_2s + 1}{T_2s}\right) = K_c \frac{s + a_g}{s} \tag{14.32}
\]

where \( T_2 = C_2R_2 \), \( a_g = 1/T_2 \), and \( K_c = -R_2/R_1 \).

Hence with PI-controllers the output value is of the aggregate of the amplified input pulse and the integral of the input value. PI-controllers are normally used for speed and current control. The addition of integral
feedback is to eliminate the steady-state error and to reduce the required forward gain.

(3) PD-Controller (proportional plus derivative action):
The aim of using PD-controller is to increase the stability of the system by improving control since it has an ability to predict the future error of the system response. In order to avoid effects of the sudden change in the value of the error signal, the derivative is taken from the output response of the system variable instead of the error signal. Therefore, D mode is designed to be proportional to the change of the output variable to prevent the sudden changes occurring in the control output resulting from sudden changes in the error signal. In addition D directly amplifies process noise therefore D-only control is not used. Fig.14.12 shows a PD-controller using Op-amplifier. The transfer function of the circuit is,

\[
\frac{v_o}{v_i} = -\frac{R_2}{R_1} (1 + sC_1R_1)
\]

(4) PID-Controller (Proportional pulse integral plus derivative action):
PID-controller has the optimum control dynamics including zero steady state error, fast response (short rise time), no oscillations and higher stability. The necessity of using a derivative gain component in addition to the PI-controller is to eliminate the overshoot and the oscillations occurring in the output response of the system. One of the main advantages of the PID-controller is that it can be used with higher order systems. PID-controller has the combination of PI and PD characteristics. Fig.14.13 shows a PID-controller using Op-amplifier. The transfer function of the circuit is,
\[
\frac{v_o}{v_i} = K \left( 1 + \frac{1}{s\tau_i} + s\tau_d \right)
\]  

(14.34)

where \( K = -\left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = -K_o, \tau_i = C_1R_1K_o \) and \( \tau_d = R_2C_1/K_o \)

The time response for the different types of controller are shown in Fig.14.14. Table 14.1 gives comparison between the characteristics of the four types of controllers.

![Time response and steady-state errors for : (a) P-controller, (b) PI-controller , (c ) PD-controller, and (d) PID-controller.](image)

Table 14.1. Comparison between the characteristics of controllers.

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-controller</td>
<td>Relatively fast , but low statistical accuracy</td>
</tr>
<tr>
<td>PI-controller</td>
<td>Relatively fast , high statistical accuracy</td>
</tr>
<tr>
<td>PD-controller</td>
<td>Very fast , but low statistical accuracy</td>
</tr>
<tr>
<td>PID-controller</td>
<td>Fast , high statistical accuracy</td>
</tr>
</tbody>
</table>
However, the most common type controller used in d.c. drive is the PI-controller. A complete closed-loop control system with both speed and current control using PI-controllers is shown in Fig.14.15.

Fig.14.15 Block diagram of thyristor speed control system with speed and current controllers (structure).

14.5.2 Mathematical Modeling of the Power Converter Units

Power converters cannot always be considered purely as simple gain as stated earlier. They usually have a delay associated with them. Therefore, each converter should be studied alone to find its transfer characteristic and its mathematical modeling in order to solve the entire control system. To understand the converter delay and find a control model to represent a specific type of converter, the main two categories of the converters used in d.c. drives namely, controlled rectifiers and d.c. chopper drives will be considered here in after.

1. Thyristor bridge converters

The thyristor bridge converter used in d.c. drives is usually provided with a.c. supply through a single-phase transformer and fast acting a.c. contactors. The d.c. output is fed to the armature of the d.c. motor. For the separately-excited motor, field is separately excited, and the field supply can be kept constant or regulated, depending on the need for the field weakening mode of operation. Here, the feedback control system with the thyristor elements is, in fact, non-linear sample-data system with variable parameters. The block diagram of such system is shown in Fig.14.16. In this case the thyristor unit is considered as sampler with a sampling period $T$ is given by
\[ T = \frac{1}{qf} \]  \hspace{1cm} (14.35)

where \( f \) = supply frequency and the factor (q) depends on the drive type. The values are: \( q = 1 \) for half-wave rectifier, \( q = 2 \) for full-wave rectifier and \( q = 6 \) for three-phase rectifier.

![Fig.14.16 Schematic diagram of a thyristor controlled d.c. motor with tachogenerator feedback.](image)

For full-wave rectifier, the static characteristic of the thyristor unit is given by the following equation (Reference 51):

\[ V_{av} = \frac{V_m}{\pi} \left( 1 - \cos(\pi \frac{V_c}{V_{cm}}) \right) \]  \hspace{1cm} (14.36)

where

- \( V_c \) = Control voltage of the triggering circuit (usually 0 to 10 V).
- \( V_{cm} \) = Maximum control (reference) voltage of the triggering circuit (10V).

Equation (14.36) is depicted on Fig.14.17, curve (1). It can be seen that the static characteristic is nonlinear and the gain of thyristor unit is not constant since \( \frac{\partial V_{av}}{\partial V_c} \) will be varied according to \( V_c \). The function \( \frac{\partial V_{av}}{\partial V_c} = f \left( \frac{V_c}{V_{cm}} \right) \) is shown in Fig.14.17, Curve (2).
The nonlinear characteristic of the power converter shown in Fig.14.17 can be approximated by a linear one. Moreover, the gain of the rectifier unit is assumed to be constant and equal to

$$K_R = \frac{\Delta V_{av}}{\Delta V_c} \quad (14.37)$$

The zero-order hold of the thyristor unit represents a memory holding average voltage until the next coming pulse from the triggering unit. Hence the sampler and hold of the thyristor unit can be replaced by a time delay of $T/2$, where $T$ is the sampling period as shown in Fig.14.18.
Hence, the dynamic of the ac-to-dc thyristor converter (rectifier) is usually described by a transfer function $G_R(s)$ with a time delay (or transportation lag) and certain gain $K_R$, i.e.

$$G_R(s) = \frac{V_a(s)}{V_{ar}(s)} = K_R e^{-\left(\frac{T}{2}\right)}$$

(14.38)

where $T = \frac{1}{qf}$ and $K_R$ = gain of the controller.

The exponential function in the above equation is approximated by rational algebraic function using Pade’s approximation so that

$$G_R(s) = \frac{V_a(s)}{V_{ar}(s)} = \frac{K_R}{1 + \frac{T}{2} s}$$

(14.39)

In this case the thyristor unit is considered as sampler with a sampling period $T = \frac{1}{qf}$, where $f$ = supply frequency and the factor $(q)$ depends on the drive type (i.e. the number of pulses), $(q = 1$ for single-phase half-wave rectifier, $q = 2$ for single-phase full-wave rectifier and corresponding by $q = 6$ for three-phase rectifier). For full-wave single-phase rectifier with 50 Hz: $T = \frac{1}{2 \times 50} = 0.01$ s. Therefore, the average delay caused by this type of converter is 5 ms as depicted in Fig.14.19.

![Fig.14.19 Time delay representation of a single-phase full-wave rectifier.](image)

For three-phase rectifier: Maximum delay = $360^\circ / 6 = 60^\circ$. In 50 Hz, this is equivalent to 20 ms/6 = 3.3 ms, therefore, the average delay caused by this type of converter is 3.3 ms /2 = 1.67 ms as depicted in Fig.14.20.

![Fig.14.20 Time delay representation of a three-phase full-wave rectifier.](image)
2. DC Chopper drives

Consider a dc-to-dc converter (chopper) with a duty cycle \( \gamma \) \( (t_{on} = \gamma T) \). According to Eq.(12.2) the average output voltage of the chopper is directly proportional to the duty cycle which is a direct function of the control voltage (usually 0 to 10 V). If \( V_{cm} \) is the maximum value of the control voltage (10 V), then the gain \( K_{cr} \) of the chopper can be expressed mathematically as

\[
K_{cr} = \frac{V_c}{V_{cm}}
\]  

When the chopper operating in the steady-state at a duty cycle \( \gamma_1 \) and the controller demands a new duty cycle \( \gamma_2 \) shortly after triggering of the chopper thyristor then there is a delay of almost \( T \) before the chopper can react. Usually, this delay is approximated as a first order lag \( \frac{1}{(1+s\tau)} \) with a time constant \( \tau = T/2 \). Thus the average delay of a chopper with a switching (control) period \( T \) is represented as in Fig.14.21.

![Fig.14.21 Time delay representation of a d.c. chopper.](image)

14.5.3 Closed-Loop Current Control with PI-Controller

As stated before that, in practice, the most type used of current controller is the proportional plus integral (PI). To analyse a drive with both speed (outer loop) and current (inner loop) controllers, it is preferred that the inner loop is analysed first. The simple block diagram of the inner loop is shown in Fig.14.22.

![Fig.14.22 Current controller (inner loop).](image)
From Eq.(14.32), the transfer function of the current controller is re-written as

\[ G_c(s) = K_c \frac{(s + a_g)}{s} \]  \hspace{1cm} (14.41)

To simplify analysis, we initially assume that the power converter is just a simple gain \( K_R \) (i.e. ignoring the time lag in the converter). Therefore the function of the power converter is to convert the input reference voltage \( V_{ar} \), for example 0-12V, to armature voltage \( V_a \), say 0-500V.

If the speed effect is ignored, we set \( E_a = 0 \) in the block diagram since \( E_a \) is directly proportional to the speed.

From the block diagram of Fig.14.19, the forward transfer function \( G(s) \) and the closed-loop transfer function \( T(s) \) of the system may be obtained as well as the feedback transfer function \( H(s) \). The general relation between these transfer functions is

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} \]

The characteristic equation of the system can be written as

\[ 1 + G(s)H(s) = 0 \]

which can be rearranged mathematically in the following form

\[ s^2 + as + b = 0 \] \hspace{1cm} (14.42)

where \( a \) and \( b \) are functions of known parameters \( (R_a, L_a) \) and unknown parameters \( (K_c, a_g) \).

Equation (14.42) is used for the design of the inner loop system and evaluation of its performance. The design specification usually requires a system dominated by a second order pole pair, with natural frequency \( \omega_n \), and damping factor \( \zeta \). Therefore, the characteristic equation can be written as

\[ s^2 + 2\xi \omega_n s + \omega_n^2 = 0 \] \hspace{1cm} (14.43)

Now, by equating the coefficients of \( s^1 \) and \( s^0 \) one can find the desired \( K_c \) and \( a_g \) for the current controller.

To find how the system is stable, we can use any linear control theory such as root locus method or Bode plot as it will be described in the following examples.
Example 14.2

A block diagram of a d.c. drive control system is shown in Fig.14.23. The drive employs a d.c. separately-excited motor has the following parameters:

\[ R_a = 0.2 \, \Omega, \quad L_a = 0.004 \, H \]
\[ \text{Inertia constant} \quad J = 5.5 \, \text{kg.m}^2 \]
\[ \text{Friction coefficient} \quad B = 0.006 \, \text{Nm} / (\text{rad} / \text{s}) \]

Design a PI - current controller using a current transducer which gives an output of 0.15 V/A. The PI-controller output \((V_{ar})\) is input to a rectifier that provides the armature circuit of the motor with the voltage \(V_a\), where \(V_a / V_{ar} = 40\). The closed-loop band width should be 300 rad /s and the damping factor \(\zeta\) should be 0.6.

![Block diagram of d.c. drive system with PI - current controller.](image)

**Solution**

The forward transfer function is

\[
G(s) = K_c \frac{(s + a_g)}{s} \times 40 \times \frac{1}{0.20 + s 0.004}
\]

\[ H(s) = 0.15 \]

The complete transfer function of the system is

\[
T(s) = \frac{G(s)}{1 + G(s)\, H(s)}
\]

The characteristic equation is

\[ 1 + G(s)H(s) = 0 \]

\[ 1 + K_c \frac{(s + a_g)}{s} \times 40 \times \frac{1}{0.20 + s 0.004} \times 0.15 = 0 \]
Simplification of the above equation yield

\[ s^2 + (50 + 1500K_c) s + 1500 K_c a_g = 0 \]

Compare this equation with the standard second order equation:

\[ s^2 + 2 \xi \omega_n s + \omega_n^2 = 0 \]

\[ 2 \xi \omega_n = 2 \times 0.6 \times 300 = 50 + 100K_c \]

\[ K_c = \frac{310}{1500} = 0.206 \]

\[ 1500 \ K_c a_g = 0.206 \times 1500a_g = 300^2 \]

\[ a_g = \frac{90000}{310} = 290 \]

The root locus of the system is shown in Fig.14.24 for clarity.

![Fig.14.24 Root locus.](image)

### 14.5.4 Closed-Loop Speed Control with PI-Controller

After analysing the current controller, as shown in the previous subsection, the speed controller is considered in the next step. The speed controller output must be related to torque, the torque must change to change the speed. The speed control (outer) loop is depicted in Fig.14.25. To start design of the controller, we make the speed controller output as the armature current reference \( I_{ar} \). This value of the current must be limited so that the armature current does not exceed the rated value for the power converter. This is usually 150% full load current of the motor. The current loop can be approximated to a second or first order system or even just a simple gain if its bandwidth is very high.
From the block diagram of Fig.14.25, the forward transfer function $G(s)$ and the closed loop transfer function $T(s)$ of the system may be obtained as well as the feedback transfer function $H(s)$. The transfer functions function of the system is obtained, as in the case of the current controller, after that the characteristic equation of the system is obtained and written in the standard quadratic equation form (Eq.14.43).

Equation (14.43) is used for the design of the outer loop system and evaluation of its performance. The design specification usually requires a system dominated by a second order pole pair, with natural frequency $\omega_n$, and damping factor $\xi$. The following example (14.3) illustrates the design procedure for a speed controller.

**Example 14.3**

A speed controller is to be designed for a 1500 rpm d.c. motor with $J = 0.01 \text{ Nm} / \text{ rad} / s^2$, $B = 0.095 \text{ Nm} / \text{ rad} / s$, using PI-controller. The current controller can be approximated to a first order lag of time constant 20 ms, and torque / back emf constant is 9 Nm /A; speed reference is provided by a 0 – 10V supply and speed measurement is from a tachogenerator giving 15 mV/rpm. Assume the armature current is limited to 5 A in control loop,

(a) Sketch the block diagram for the control system.

(b) Find the gain $K_c$ and the factor $a_g$ for the PI-speed controller to provide closed loop control of bandwidth 10 rad/s, damping factor = 0.7. Sketch the root locus.

**Solution**

(a) The current controller transfer function is

$$G_R(s) = \frac{K_{pc}}{1+s\tau} = \frac{1}{1 + 20 \times 10^{-3}s} = \frac{50}{s + 50}$$

For the tachogenerator, $K_t = 0.015 \text{ V/rpm} = 0.1432 \text{ V} / \text{ rad} / s$. 

![Fig.14.25 Speed controller loop.](image)
The block diagram for the control system is shown in Fig. 14.26.

![Block Diagram](image)

Fig. 14.26.

(b) The characteristic equation of the system in Fig. 14.26 is

\[
1 + K_c \frac{(s + a_g)}{s} \times \frac{50}{s + 50} \times \frac{9}{0.01s + 0.095} \times 0.1432 = 0
\]

\[s^3 + 59.5 s^2 + 475.5 s + K_c(s + a_g)6444 = 0\]  \( (A) \)

The reference equation has \( \omega_n = 10, \xi = 0.7 \) and an unknown pole \( \alpha \).

Hence,

\[(s + \alpha)(s^2 + 2\xi \omega_n s + \omega_n^2) = 0\]

or \[s^3 + (17 + \alpha) s^2 + (100 + 17\alpha)s + 100 \alpha = 0\]  \( (B) \)

Equating coefficients of Eq.\((A)\) and Eq.\((B)\) :

\[\alpha = 42.5, \quad K_c = 0.0567, \quad a_g = 11.63\]

and the closed loop poles are \(-49, -11.8, -9.5 \) and \(0\). The root locus is shown in Fig. 14.27.

![Root Locus](image)

Fig. 14.27 System root locus.
14.5.5 Closed-Loop Speed and Current Control with PI Controller
Simplified Steady-State Analysis

Fig. 14.28 shows a typical block diagram of d.c. drive system with both speed and current control using PI-controllers. The d.c. drive closed-loop system employs ac-to-dc converter using thyristor or any other power semiconductor devices elements is a standard full-wave fully-controlled rectifier bridge.

Fig. 14.28  Block diagram of thyristor speed control system with speed and current controllers (work diagram).

The thyristor bridge converter is provided with a.c. supply through a single-phase transformer and fast acting ac contactors. The d.c. output is fed to the armature of the d.c. motor. The field is separately excited, and the field supply can be kept constant or regulated, depending on the need for the field weakening mode of operation. Here, the feedback control system with the thyristor elements is, in fact, non-linear sample-data system with variable parameters. In this case the thyristor unit is considered as sampler with a sampling period \( T = 1/q f \), where \( f \) = supply frequency and the factor \( q \) depends on the drive type, \( q = 1 \) for half-wave rectifier, \( q = 2 \) for full-wave rectifier) as illustrated in subsection 14.5.2. If either speed or current control is used then the above system is reduced to Fig. 14.29.

Let \( G = \) open loop transfer function.

\[
T(s) = \frac{G(s)}{1 + G(s)H(s)}
\]

The over all transfer function of the system is
The steady-state operation of the system can be obtained by applying the final value theorem by setting \( s = 0 \), hence the block diagram of the system will reduced to more simple one as depicted in Fig.14.30.

\[
\frac{N}{V_r} = \frac{K_0}{K_0 + Kt + St_m + s^2T_aT_m + 1} \tag{14.44}
\]

where

\[ K_0 = \frac{K_R K_A}{K_m} \]

Note: this analysis is only valid if the effect of the back emf of the motor is neglected.

---

**Example 14.4**

The thyristor speed control of a d.c. separately-excited motor is drawn in Fig.14.31. The d.c. motor has the following data:

- Rated armature voltage: \( V_{a(ay)} = 180 \) V.
- Rated armature current: \( I_a = 8.7 \) A
- Rated speed: \( n = 880 \) rpm
Armature resistance \( R_a = 3.15 \Omega \)

The thyristor unit has gain \( K_R = \frac{V_{a(\text{av})}}{V_x} = 0.2 \)

The tachogenerator has a gain \( K_T = \frac{V_T}{\omega} = 0.2 \) Volt / rad/s

Machine constant \( K\Phi = 1.655 \) V/rad/s

It is required to solve:
(a) The drop of the speed \( \Delta \omega \) of the d.c. motor without speed controller (open-loop operation) at rated armature current and at rated voltage.
(b) The drop of the speed \( \Delta \omega \) for the same armature current in (a) considering the effect of the speed controller and the feedback control if the reference voltage \( V_r = 13.55 \) V.

Solution

For steady-state operation, the speed of d.c. motor is given by Eq.(11.25) as
\[
\omega = \frac{V_{a(\text{av})}}{K\Phi} - \frac{R_a I_a}{K\Phi} = \omega_o - \Delta \omega
\]

The no load speed is
\[
\omega_o = \frac{V_{a(\text{av})}}{K\Phi}
\]

The drop in speed is
\[
\Delta \omega = \frac{R_a I_a}{K\Phi} = \frac{3.15 \times 8.7}{1.655} = 16.5 \text{ rad/s}
\]
Considering the speed controller effect:

The closed-loop block diagram of the d.c. motor with speed controller only is shown in Fig.14.32.

![Fig.14.32](image)

Now

\[ \Delta V = V_r - V_T \]

\[ V_x = K_C \cdot \Delta V \]

\[ V_{a(av)} = K_R \cdot V_x = K_C K_R (V_r - V_T) \]

\[ V_T = K_T \cdot \omega \]

\[ T = K_\Phi I_a = (1/K_m) \cdot I_a \]

Since \( K_m = 1/K_\Phi = 1/1.655 = 0.606 \text{ rad/V.s} \), in this case,

\[ \omega = \frac{V_{a(av)}}{K_\Phi} - \frac{R_a}{K_\Phi} I_a \]

\[ \omega = \frac{K_C K_R (V_r - K_T \cdot \omega)}{K_\Phi} - \frac{R_a}{K_\Phi} I_a \]

\[ \omega = \frac{K_C K_R}{K_\Phi} \frac{V_r}{K_\Phi} - \frac{K_C K_R K_T \cdot \omega}{K_\Phi} - \frac{R_a}{K_\Phi} I_a \]

\[ \omega \left( 1 + \frac{K_C K_R K_T}{K_\Phi} \right) = \frac{K_C K_R}{K_\Phi} \frac{V_r}{K_\Phi} - \frac{R_a}{K_\Phi} I_a \]

\[ \omega \left( 1 + K_C K_R K_T K_m \right) = K_m K_C K_R \frac{V_r}{K_\Phi} - K_m R_a I_a \]

Let \( K_o = K_C K_R K_T K_m \)

\[ \therefore K_o = 10 \times 30 \times 0.2 \times 0.606 = 36.25 \]
The change in speed with closed-loop controllers is

\[
\Delta \omega' = \frac{K_m R_a I_a}{1 + K_o} = \frac{0.606 \times 3.15}{1 + 36.25} \times 8.7 = 0.445 \text{ rad/s}
\]

or

\[
\Delta n' = \frac{0.445 \times 60}{2\pi} = 4.25 \text{ rpm}
\]
14.3 A d.c. separately-excited motor with voltage control by thyristor switching is used as a drive motor in a close-loop system with tachometric feedback. Sketch such a system in 'block-box' form. If the system is operating at low speed, what would be the effect the suddenly increasing the reference signal on the motor error signal, the thyristor firing-angle, the motor voltage and the speed? (use Matlab simulation if you could, sketch the type of speed response versus time that you would consider acceptable and reasonable in such a system). What are the advantages and disadvantages of using thyristor chopper and thyristor bridge rectifier for voltage control of a d.c. motor?

14.4 A separately-excited d.c. motor with machine constant $K_\phi = 0.54$ is controlled by a closed-loop system, as shown in Fig.14.30, including a power converter which has a gain of 100. The armature resistance and inductance of the motor are 0.7 $\Omega$ and 0.1 H respectively. The armature inertia $J = 0.05$ kgm$^2$ and frictional damping $B$ is negligible. The motor is connected through a 10:1 gearbox to a load which has inertia 5 kg.m$^2$ and torque which is proportional to $(speed)^2$: $T = 0.01 \omega^2$.

The motor shaft speed is sensed by a tachogenerator, which is a precision permanent magnet d.c. generator producing 9.5 V/1000 rpm. The error signal which is the input to the power converter is derived by subtracting the tachogenerator output from a reference voltage. By approximating the load torque characteristic about an operating point of 100 rad /s, derive a closed-loop transfer function for the system and calculate its natural frequency and damping ratio. The block diagram of the system is shown in Fig.14.33.

![Fig.14.33.](image)

$[\text{Ans: } \frac{V_o(s)}{V_{ref}(s)} = \frac{G(s)}{1 + G(s)} = \frac{49.14}{s^2 + 7.2s + 50.83}, \omega_n = 7.13 \text{ rad/s}, \xi = 0.5]$}

14.5 A conveyor is to be speed controlled by a d.c. drive using two d.c. motors coupled on a common shaft. The motor ratings are 300 kW, 600 V, 1250 rpm and 600 kW, 500 V, 1250 rpm. Each motor has its own thyristor converter match its rating, employing conventional current loops. A tachogenerator is fitted on the shaft. Suggest a method or methods of control to provide:
(a) Overall speed control.
(b) Correct load sharing.

14.6 A 30 kW, 240 V, 4000 rpm shunt-excited d.c. motor is by three-phase half-controlled rectifier of gain $K_R = 200$. The armature resistance of the motor $R_a = 0.045 \, \Omega$ and the armature inductance is 0.72 H. The machine constant $K \phi = 0.628 \, \text{V.s/rad}$.

(a) Obtain the open-loop transfer functions $\omega(s) / V_r(s)$ and $\omega(s) / T_L(s)$ for the motor.
(b) If the speed reference voltage $V_r$ is set to 1.0 V and the torque is reduced by 40% of its rated value, calculate the speed of the motor under these conditions.

[Ans: $n = 2988 \, \text{rpm}$]

14.7 Repeat problem 14.6 if a tachogenerator of amplification factor $K_t = 3 \, \text{mV.s/rad}$ is used to control the speed of the motor in closed-loop mode of operation.

[Ans: $n = 1528 \, \text{rpm}$]

14.8 Fig.14.34 shows d.c. motor drive system block diagram with PI-current controller. The error signal from $I_a$ loop is fed into the PI-controller. The output of the PI-controller, $V_{ar}$, is fed to power converter / d.c. motor system. The current control loop is taking into consideration the effect of the mechanical time constant $T_m$.

(a) Show that the system can be reduced to the block diagram shown in Figs.14.35 and 14.36.
(b) Obtain expressions for the overall gain $K$, $\omega_n$ and $\xi$ in terms of the mechanical and electrical time constants ($\tau_m$ and $\tau_a$) of the system,
(c) Given that $\omega_n = 10 \, \text{rad/s}$, $\xi = 0.5$ and $a_g = 18 \, \text{rad/s}$, sketch the root locus as $K$ varies from 0 to infinity, given that $K_c = 0.095$.
(d) Calculate the values of the closed-loop poles when $K = 2$ and show that the closed-loop transfer function is given by

$$T(s) = \frac{200(s + 18)}{(s + 20)(s + 19)}$$

Note: $\tau_a = \frac{L_a}{R_a}$, $\tau_m = J \frac{R_a}{K_m^2}$

$k_m$ = Motor constant in SI unit (V.s/rad), $K_i$ = Current transducer gain.
[Ans: (a) \( K = \frac{K_R K_c K_i}{R_a} \) \( \tau_m \), \( \omega_n = \sqrt{\frac{1}{\tau_a \tau_m}} \),

\( \xi = \frac{\tau_m}{4\tau_a} \), (d) \(-20, -190\)]
14.9 A separately-excited d.c. motor drives a load with equivalent inertia referred to the motor equal to $J$. By considering the effect of the armature circuit inductance $L_a$, show that the response of the current is affected by both the electrical and mechanical time constants defined in problem 14.9.

Hint: Use the fundamental electrical and mechanical equation of the motor to solve for the current response.

14.10 In a closed-loop speed control of a separately-excited motor, derive the transfer function ($\omega_m/\nu$) for the motor by considering the load as a pure inertia. Use the superposition theory to show that, by examine the effect of load torque $T_m$, the speed response will be oscillatory if the electrical time constant of the motor $\tau_a > \tau_m/4$, hence derive the frequency of oscillation in terms of these time constants. The open loop block diagram of the system is shown in Fig.14.37.

Fig.14.37 Open-loop block diagram of a separately-excited d.c. motor.

$$[\text{Ans: } TF = \frac{\omega(s)}{V_a(s)} = \frac{1/K\phi}{\tau_a\tau_m s^2 + \tau_m s + 1}, \text{ undamped oscillatory frequency}$$

$$\omega_n = \frac{1}{\sqrt{\tau_a\tau_m}}$$

14.11 If the motor in problem 14.11 is supplied from an electronic amplifier of linear gain $K$ with a close-loop speed control system, show how will be the effect of adding the amplifier on the speed response. Use the transfer function obtained in problem 14.11.

14.12 Figure 14.38 shows the block diagram of a speed control system of a d.c. motor where $V_{in}$ is the reference input voltage, $\omega$ is the output speed and overall feedback loop with constant $K_t$ represents the tachogenerator feedback.

(a) Derive the closed-loop transfer function ($\omega/V_{in}$) in terms of the system constants.
$V_{in}$ is now adjusted to give a steady output speed of 1300 rpm. Due to fault, tachogenerator is reduced to zero and this increases the steady output speed to 1500 rpm. By what factor must $K_1$ be changed to restore the speed to 1300 rpm.

(b) Discuss the effect of this on the transient behaviour of the system.

Fig.14.38.

14.13 A d.c. motor is to be controlled by terminal voltage variation using single-phase fully-controlled rectifier. Sketch a diagram of this arrangement for open-loop control and point out the advantages and disadvantages compared with, say, d.c. chopper control.

The control loop is now closed using a tachogenerator to give negative feedback. Sketch the system in block-diagram form, defining transfer functions or transfer characteristics for each block. Discuss the inherent difficulties of analysing such a system by linear control theory. How would you approach the analysis of such a system?

The response of motor shaft speed to a step increase of control signal in a typical case is given in Fig.14.39. What do you deduce from this about the nature of the system?

Fig.14.39.