Question 1:  

Objectives: Understanding the concepts of power system stability.

Mark (✓) for the correct statement and (X) for false statement of the following:

1. The transient stability can be lost as a result of short circuits, whereas less severe faults, such as disconnection of generation or a transmission lines, do not lead to the loss of transient stability.

2. An early fault clearing means better chances of maintaining system stability.

3. In the equal-area criterion the accelerating area $A_1$ and decelerating area $A_2$ are determined, then if $A_2 < A_1$ the system is stable.

4. The longest duration of disturbance may result in instability of the system.

5. The use of automatic reclosing circuit breakers improves system stability.

6. Equal area criterion method is used to solve the steady-state stability problems.

7. Connecting more transmission lines in parallel may improve the system stability.

8. The synchronizing power coefficient $P_s$ has strong effect on the system natural frequency of oscillation $\omega_n$.

9. Equations describing the steady-state stability can be linearized.

10. For the figure shown below which shows the solution for rotor angle $\delta$, the system is likely to be stable.

![Graph of rotor angle $\delta$ over time]
**Question 2:**

**Objectives:** Evaluation of the steady–state stability of a power system due to small perturbation.

A 50-Hz., 500MVA synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 6 MJ/MVA and a per unit damping coefficient of $D = 0.10$. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure 1. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator excitation voltage is $E' = 1.25$ per unit with $\delta = 30^\circ$ and is delivering a real power of 0.8 per unit to the infinite bus at a voltage of $V = 1.0 /0^\circ$ per unit.

(a) Write the linearized swing equation for this power system and compute the synchronizing power coefficient $P_s$.

(b) The generator is operating in the steady state at $\delta_0 = 30^\circ$ when the input power is increased by a small amount $\Delta P = 0.12$ per unit. The generator excitation and the infinite bus voltage are the same as before. Find the equations describing the generator rotor angle and frequency.

Note: See Laplace transforms on page 3.

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**Question 3:**

**Objectives:** Evaluation of the transient stability of a power system due to large perturbation.

A generator is connected by a double line to an infinite bus, the voltage of which is $V = 1.0$ pu as shown in Fig.2. Per-unit values of reactances and voltages are indicated in the figure. A three-phase short circuit occurs at the point P. The circuit breakers A and B open simultaneously and remain open. The mechanical power supplied to the generator before fault is $P_m = 1.0$ pu.

(a) Determine the electrical powers $P_{e1}$, $P_{e2}$ and $P_{e3}$ before, during and the fault.

(b) Draw on same graph power angle curves for $P_{e1}$, $P_{e2}$ and $P_{e3}$.

(c) Calculate the angles $\delta_0$, $\delta_1$ and $\delta_{max}$.

(d) Write equations for $A_1$ and $A_2$ for critical clearing angle $\delta_{cr} = 55$
Question 4: (15 Marks)

Objectives: Solving the swing equation using direct methods of solution

A 3-phase, 100Hz, 60 MVA synchronous generator with $H = 4.5\text{MJ/MVA}$ operating at steady state with input and output power equal to 0.7 p.u and rotor displacement angle $\delta = 30^\circ$ with respect to infinite bus. Due to occurrence of a fault, the output power–angle relation is given by $P_e = 1.0 \sin \delta$. Assuming that the input power remains constant, determine three points of the swing curve using the step-by-step (point-by-point) method, take the time interval $\Delta t = 0.05$ second.

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**Laplace Transform Analysis**

\[
\Delta \delta(s) = \frac{(s + 2\zeta \omega_n) \Delta \delta_0}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]
\[
\Delta \omega(s) = \frac{\omega_n \Delta \delta_0}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]
\[
\Delta \delta(t) = \frac{\Delta \delta_0}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n t + \theta), \quad \theta = \cos^{-1} \zeta
\]
\[
\Delta \omega(t) = \frac{\omega_n \Delta \delta_0}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n t)
\]
\[
\delta(t) = \delta_0 + \Delta \delta(t), \quad \omega(t) = \omega_0 + \Delta \omega(t)
\]

\[
\delta = \delta_0 + \frac{\pi f_0 \Delta P}{H \omega_n^2} \left[1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n t + \theta)\right]
\]
\[
\omega = \omega_0 + \frac{\pi f_0 \Delta P}{H \omega_n \sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n t
\]
Question 1: (10 Marks)

1. ✓ (true) [IM]
2. ✓ (true) [IM]
3. ✗ (false) [IM]
4. ✓ (true) [IM]
5. ✓ (true) [IM]
6. ✗ (false) [IM]
7. ✓ (true) [IM]
8. ✓ (true) [IM]
9. ✓ (true) [IM]
10. ✗ (false) [IM]
Question 2: (15 Marks)

Solution

The transfer impedance of the network:

\[ X_T = 0.2 + j0.158 + \frac{0.8 \times 0.8}{0.8 + 0.8} \]

\[ = 0.758 \text{ p.u.} \]

\[ P_e = \frac{|E||V|}{X_T} \sin \delta . = \frac{1.25 \times 1.0 \sin \delta}{0.758} \]

\[ P_s = \left. \frac{dP_e}{d\delta} \right|_{\delta = \delta_0} = 1.649 \cos \delta_0 = 1.649 \times 0.866 = 1.428 \]  

\[ \omega_n = \sqrt{\frac{\pi f_o P_s}{H}} = \sqrt{\frac{3.14 \times 50 \times 1.428}{6}} = 6.112 \text{ rad/sec.} \]

\[ \xi = \frac{D}{2} \frac{\pi f_o}{H P_s} = \frac{0.1}{2} \sqrt{\frac{3.14 \times 50}{6 \times 1.428}} = 0.214 \]

\[ \omega_d = \omega_n \sqrt{1-\xi^2} = 6.112 \sqrt{1-(0.214)^2} = 5.97 \text{ rad/sec.} \]

\[ \Theta = \cos^{-1} \xi = \cos^{-1} 0.214 = 77.64^\circ. \]

The linearized swing equation is given by:

\[ \frac{d^2 \Delta s}{dt^2} + 2 \xi \omega_n \frac{d\Delta s}{dt} + \omega_n^2 \Delta s = 0 \]

Hence

\[ \frac{d^2 \Delta s}{dt^2} + 2 \times 6.15 \frac{d\Delta s}{dt} + 37.35 \Delta s = 0. \]

[4M]
(b) For a change in $\Delta P = 0.12$ pu., this is a small change. Equation of system response to small power change are:

$$\delta = \delta_0 + \frac{180f_0 \Delta P}{H \omega_n^2} \left[ 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin (\omega_n t + \theta) \right]$$

$$\omega = \omega_0 + \frac{180f_0 \Delta P}{H \omega_n \sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_n t.$$  

and

$$f = \frac{\omega_0}{2\pi} + \frac{180f_0 \Delta P}{H^2 \omega_n \sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_n t.$$

Using the above equations we get:

$$\delta = 30 + \frac{180 \times 50 \times 0.12}{6 \times (6.112)^2} \left[ 1 - \frac{1}{\sqrt{1-0.976^2}} e^{-0.214 \times 6.112}\sin (5.97t + 77.6^\circ) \right]$$

$$= 30 + 4.81 \left[ 1 - \frac{1}{0.976} e^{-1.3t} \sin (5.97t + 77.6^\circ) \right]$$

$$\delta = 34.81 - 1.024 e^{-1.3t} \sin (5.97t + 77.6^\circ). \quad [4M]$$

$$f = 50 + \frac{180 \times 50 \times 0.12}{2 \times 3 \times 4 \times 6 \times 6.112 \times 0.976} e^{-1.3t} \sin 5.97t$$

$$= 50 + 0.083 e^{-1.3t} \sin 5.97t$$

or

$$f = 50 + 4.8 e^{-1.3t} \sin 5.97t. \quad \text{when } 180 \Rightarrow \pi.$$  

$$[4M]$$

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Question 3: (10 Marks)

Solution:

Electrical power of the generator in transient:

\[ P = \frac{|E||V|}{X_T} \sin \delta \]

(a) Before fault:

\[ X_{T1} = 0.3 + \frac{(0.15+0.15)(0.15+0.15)}{0.15+0.15+0.15+0.15} + 0.1 = 0.7 \text{ pu} \]

\[ P_{e1} = \frac{1.3 \times 1.0}{0.7} \sin \delta \]

\[ P_{e1} = 1.86 \sin \delta \]

During fault:

\[ X_{T2} = 0.375 + 0.325 + \frac{0.375 \times 0.315}{0.05625} = 2.86 \text{ pu} \]

\[ P_{e2} = \frac{(1.3)(1.0)}{2.86} \sin \delta = 0.453 \sin \delta \]

- 4 -
(a) After fault: Circuit breakers A & B opened

\[ X_{T3} = 0.3 + 0.6 + 0.1 = 1.0 \] \[ \begin{array}{ccc}
0.3 & 0.6 & 0.1 \\
E & & V \\
\end{array} \]

\[ \delta \cos \theta_3 = \frac{(1.3)(1.0)\sin \delta}{1} = 1.3 \sin \delta \]  \[\text{[4 M]}\]

(b) We plot the power angle curve

(c) \( \delta_0 \) is obtained with \( P_{e1} \):

\[ 1.0 = 1.86 \sin \delta_0 \]

\[ \sin \delta_0 = \frac{1}{1.86} \]

\[ \Rightarrow \delta_0 = 32.5^\circ = 0.567 \]

\( \delta_1 \) is obtained with \( P_{e3} \):

\[ 1.0 = 1.3 \sin \delta_1 \]

\[ \sin \delta_1 = \frac{1}{1.3} \approx 0.877 \]

\[ \delta_1 = 50.3^\circ \approx 0.877 \text{ rad.} \]

\[ \delta_{\text{max}} = 180^\circ - \delta_1 = 129.7^\circ = 2.264 \text{ rad.} \]  \[\text{[2 M]}\]

(d) For \( \delta_{\text{cr}} = 55^\circ = 0.959 \)

\[ A_1 = \int_{\delta_e}^{\delta_{cr}} (P_m - P_{e2}) d\delta = \int_{0.567}^{0.959} (1 - 0.453 \sin \delta) d\delta \]

\[ = 0.959 - 0.567 + 0.453 \cos 55^\circ 55^\circ = 0.453 \cos 30^\circ \]

\[ = 0.392 + 0.529 - 0.392 = 0.528 \]

\[ A_2 = \int_{\delta_{\text{cr}}}^{\delta_{\text{max}}} (P_{e3} - P_m) d\delta = \int_{0.959}^{2.264} (1.3 \sin \delta - 1) d\delta \]  \[\text{[2 M]}\]
**Question 4:**

**Solution**

\[ M \text{ (in pu)} = \frac{C(pw)H}{180f} = \frac{1.0 \times 4.5}{180 \times 5.0} = 2.5 \times 10^{-4} \]

\[ \Delta \omega_n = \frac{\Delta t}{M} P_{a(n-1)} = \frac{0.05}{2.5 \times 10^{-4}} P_{a(n-1)} = 200 P_{a(n-1)} \]

\[ \Delta \delta_n = \frac{(\Delta t)^2}{2M} P_{a(n-1)} + (\Delta t) \omega_{n-1} \]

\[ = \frac{(0.05)^2}{2 	imes 2.5 \times 10^{-4}} P_{a(n-1)} + (0.05) \omega_{n-1} \]

**Step 1:**

\[ t = 0^+ \quad \delta = 30^\circ \Rightarrow \delta_c \]

\[ P_{c(0)} = P_{max} \sin \delta = 1.0 \sin 30 = 0.5 \text{ p.u.} \]

but \( P_m = 0.7 \)

\[ P_{a(0)} = 0.7 - P_{c(0)} = 0.7 - 0.5 = 0.2 \text{ p.u.} \] \[ \] \[ \text{[4M]} \]

**Step 2**

\[ \Delta \omega = 200 \times 0.2 = 4\omega \]

\[ \text{At } t = 0^- \quad \omega_{n-1} = \frac{d\delta_c}{dt} = 0 = \omega_c \quad \text{(no change in } \delta) \]

\[ \Delta \delta_1 = 5 P_{a(0)} + 0.05 \omega_c = 5 \times 0.2 + 0.05 \times 0 = 1^\circ \]

\[ \delta_1 = \delta_c + \Delta \delta_1 = 30 + 1 = 31^\circ \]

\[ \text{At } t = t_1 = t_0 + \Delta t = 0 + 0.05 = 0.05 \text{ sec} \quad \delta = \delta_1 = 31^\circ \]

\[ P_{c(1)} = 1.0 \sin \delta_1 = 1.0 \sin 31^\circ = 0.51 \]

\[ P_{a1} = 0.7 - 0.51 = 0.19 \text{ p.u.} \]

\[ \Delta \omega_1 = 200 P_{a1} = 200 \times 0.19 = 38 \]

\[-6-\]
\[ \omega_1 = \omega_0 + \frac{\Delta t}{M} Pa_0 = 0 + 200 \times 0.2 = 40 \]
\[ \Delta \delta_2 = 5Pa_1 + 0.05\omega_1 = 5 \times 0.19 + 0.05 \times 40 = 2.95 \]
\[ \delta_2 = \delta_1 + \Delta \delta_2 = 31 + 2.95 = 33.95^\circ = 33^\circ48' \]

**Step 3.**

At \( t = 0.05 + 0.05 = 0.1 \)

\[ Pe_2 = 1 \cdot \sin \delta_2 = \sin 33.95^\circ = 0.56 \]
\[ Pa_2 = 0.7 - 0.56 = 0.14 \]
\[ \Delta \omega = 200 Pe_2 = 200 \times 0.14 = 28 \]
\[ \omega_2 = \omega_1 + \frac{\Delta t}{M} Pa_1 = 40 + \frac{0.05}{2.5 \times 10^4} \times 0.14 = 68 \]
\[ \Delta \delta_3 = 5Pa_2 + 0.05\omega_2 = 5 \times 0.14 + 0.05 \times 68 = 4.1 \]

\[ \therefore \delta_3 = \delta_2 + \Delta \delta_3 = 33^\circ48' + 4.1 = 38^\circ1' \]

**Table for the three points is:**

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<td>33.95° = 34°</td>
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**Dr. M.T. Lazim**

M. Lazim

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