

## Example

Example 1: let UD be the real number

$$P(x,y) = x * y = 0$$

What is the truth value of the following:

$$\forall x \forall y P(x,y) : \text{false}$$

$$\forall x \exists y P(x,y) : \text{true}$$

$$\exists x \forall y P(x,y) : \text{true}$$

$$\exists x \exists y P(x,y) : \text{true}$$

Example 1: let UD be the real number

$$P(x,y) = x / y = 1$$

What is the truth value of the following:

$$\forall x \forall y P(x,y) : \text{false}$$

$$\forall x \exists y P(x,y) : \text{true}$$

$$\exists x \forall y P(x,y) : \text{false}$$

$$\exists x \exists y P(x,y) : \text{true}$$

For example, in  $\forall x \exists y (x + y = 0)$ , you would loop through ALL the elements in the domain for x, searching for AT LEAST one element for y that satisfies the statement.

In  $\exists x \forall y Q(x,y)$ , you would loop through the domain, testing every x until you find ONE x that satisfies the statement for ALL y

English	First-Order
At least one x is P	$\exists xP(x)$
All x are P	$\forall xP(x)$
Some x are P	$\exists xP(x)$
Not all x are P	$\exists x\neg P(x)$
No x are P	$\forall x\neg P(x)$

$$\neg(\forall xP(x)) \equiv \exists x(\neg P(x))$$

$$\neg(\exists xP(x)) \equiv \forall x(\neg P(x))$$

$$\forall x\forall yQ(x, y) \equiv \forall y\forall xQ(x, y)$$

$$\exists x\exists yQ(x, y) \equiv \exists y\exists xQ(x, y)$$