Chapter 2

1

VECTORS

1

Lecture 03

2

Vectors Multiplication

- The Scalar (Dot) Product

2

Scalar Product of Two Vectors

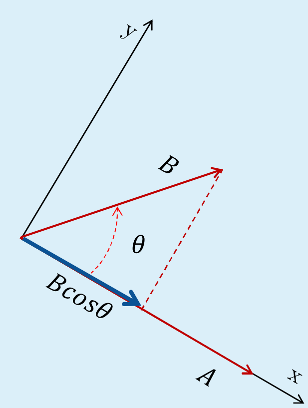
The scalar product of two vectors is defined as.

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\,\mathrm{Cos}\,\theta$$

- It is also called the dot product.

Scalar Product of Two Vectors

- θ is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn starting at the same point.
- The scalar product of any two vectors is a scalar quantity.



Scalar Product, cont

5

The scalar product is commutative.

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

The scalar product obeys the distributive law of multiplication.

$$\vec{A}\cdot(\vec{B}+\vec{C}) = (\vec{A}\cdot\vec{B}) + (\vec{A}\cdot\vec{C})$$

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Dot Products of Unit Vectors

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$$\hat{\imath}\cdot\hat{\imath} = (1)(1)cos0^{\circ} = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = (1)(1)cos90^{\circ} = 0$$

- In general:

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = 0$$

- Using component form with vectors:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\Rightarrow \quad \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

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Dot Products of Unit Vectors

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Special cases:

$$\vec{A}\cdot\vec{B} = \begin{cases} AB; If\ they\ are\ parallel\ ;\ \theta = 0^{\circ} \\ -AB\ ;\ If\ they\ are\ anti-parallel\ ;\ \theta = 180^{\circ} \\ 0\ ; If\ they\ are\ perpendicular\ ;\ \theta = 90^{\circ} \end{cases}$$

$$\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

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The scalar product-1

Sunday, 13 June, 2021 16:12

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
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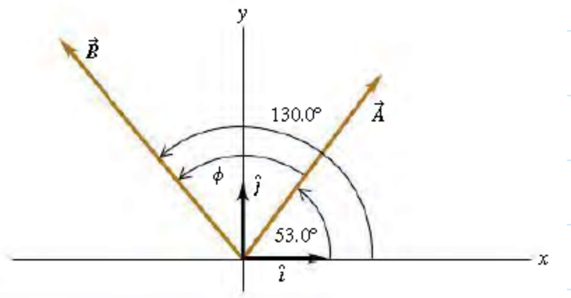
Find the scalar product of the two vectors shown in the figure. The magnitudes of the vectors are $A = 4$ and $B = 5$.



$$\vec{A} \cdot \vec{B} = AB\cos\theta = 4 \times 5 \times \cos(130° - 53°) = 4.5$$

The scalar product-2

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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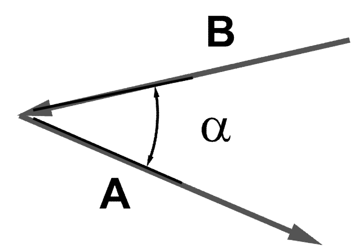
The figure shows two vectors lying in the xy plane, if $|\vec{A}| = 6$, $|\vec{B}| = 5$ and $\alpha = 40^{\circ}$. Determine the scalar product of them.



Solution

$\vec{A} \cdot \vec{B} = AB\,cos\,\theta$

Where θ is the angle between the two vectors when they are drawn starting at the same point;

$\theta = 180 - \alpha = 180 - 40 = 140°$

$\vec{A} \cdot \vec{B} = 6 \times 5 \times cos\,1\,40° = -23$

The scalar product-3

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [x] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Vectors $\vec{A}$ and $\vec{B}$ have magnitudes of 3 units and 4 units, respectively.

- What is the angle between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A}\cdot\vec{B}=0$
- What is the angle between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A}\cdot\vec{B}=12$
- What is the angle between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A}\cdot\vec{B}=-12$

Solution:

$$\vec{A}\cdot\vec{B}=AB\cos\theta=(3)(4)(\cos\theta)=0$$

$$\Rightarrow\ \theta=\cos^{-1}0=90^\circ$$

The vectors are perpendicular; $\theta=90^\circ$

Solution

$$\vec{A}\cdot\vec{B}=AB\cos\theta=(3)(4)(\cos\theta)=12$$

$$\Rightarrow\ \theta=\cos^{-1}1=0^\circ$$

The vectors are in the same direction (parallel); $\theta=0^\circ$

Solution

$$\vec{A}\cdot\vec{B}=AB\cos\theta=(3)(4)(\cos\theta)=-12$$

$$\Rightarrow\ \theta=\cos^{-1}(-1)=180^\circ$$

The vectors are in opposite directions (anti-parallel); $\theta=180^\circ$

The scalar product-4

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The vectors $\vec{A}$ and $\vec{B}$ are given by: $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = -\hat{\imath} + 2\hat{\jmath}$. Determine the scalar product $\vec{A} \cdot \vec{B}$.

Solution

The components of the two vectors are:

$A_x = 2\ ;\ A_y = 3\ ;\ B_x = -1\ ;\ B_y = 2$

The scalar product is:

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (2 \times^{-} 1) + (3 \times 2) = 4$

Angle between two vectors using dot products-1

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The vectors $\vec{A}$ and $\vec{B}$ are given by: $\vec{A} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{B} = -2\hat{\imath} + 3\hat{k}$. Find the angle between the directions of two vectors.

Solution

The components of the two vectors are:

$$A_x = 3\,;\ A_y = -4\,;\ A_z = 0\,;\ B_x = -2\,;\ B_y = 0\,;\ B_z = 3$$

The magnitude of $\vec{A}$ is:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{3^2 + (-4)^2 + 0^2} = 5$$

And the magnitude of $\vec{B}$ is:

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-2)^2 + 0^2 + 3^2} = 3.6$$

The scalar product is:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = (3 \times^{-} 2) + (^{-}4 \times 0) + (0 \times 3) = {}^{-}6$$

The angle between the directions of two vectors is included in the definition of their scalar product:

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

Substituting the results from parts (a) and (b) into the definition of the scalar product yields:

$$-6 = (5)(3.6)\cos\theta$$

$$\Rightarrow\ \theta = \cos^{-1}\left(\frac{-6}{(5)(3.61)}\right) = 109°$$

Angle between two vectors using dot products-2

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Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

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Given the vectors $\vec{A} = 3\hat{\imath} - \hat{\jmath} + 5\hat{k}$and $\vec{B} = x\hat{\imath} + \hat{\jmath} + 2\hat{k}$.

(A) Find the value of (x) that makes the two vectors perpendicular to each other.

(B) Find the angle between $\vec{A}$ and the positive x-axis.

Solution

Since $\vec{A} \perp \vec{B}$

$$\Rightarrow \vec{A} \cdot \vec{B} = AB \cos 90° = 0$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = 0$$

$$(3x) + (-1 \times 1) + (5 \times 2) = 0$$

$$\Rightarrow x = -3$$

Solution

$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \hat{\imath}}{|\vec{A}||\hat{\imath}|}\right) = \cos^{-1}\left(\frac{3 \times 1 + {}^{-}1 \times 0 + 5 \times 0}{1 \times \sqrt{3^2 + (-1)^2 + 5^2}}\right) = 59.5°$$

Angle between two vectors using dot products-3

Friday, 29 January, 2021 21:33

The vectors $\vec{A}$ and $\vec{B}$ are given by: $\vec{A} = 3\hat{\imath} - 4\hat{\jmath} + \hat{k}$ and $\vec{B} = -2\hat{\imath} + 3\hat{k}$.

- Find the magnitude of the two vectors.
- Determine the scalar product $\vec{A} \cdot \vec{B}$.
- Find the angle between the directions of two vectors.
- Find the angle between $\vec{A}$ and the positive x-axis.

Answers:

$$|\vec{A}| = \sqrt{26}, |\vec{B}| = \sqrt{13}$$

$$\vec{A} \cdot \vec{B} = -3$$

$$\theta = 99.4°$$

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