Chapter 2

1

VECTORS

1

Lecture 04

2

Vectors Multiplication

- The Vector (Cross) Product

2

The Vector Product

3

- There are instances where the product of two vectors is another vector.
  - Earlier we saw where the product of two vectors was a scalar.
- The vector product of two vectors is called the cross product.

3

The Vector Product Defined

4

The vector (cross) product of $\vec{A}$ and $\vec{B}$ is defined as a third vector:

$$\vec{C} = \vec{A} \times \vec{B}$$

The magnitude of $\vec{C}$:

$$|\vec{C}| = |\vec{A}||\vec{B}|sin\theta$$

θ is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn starting at the same point.

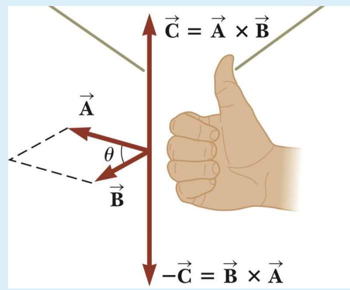
4

More About the Vector Product

5

The direction of $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.

The best way to determine this direction is to use the right-hand rule (R.H.R).



5

Properties of the Vector Product

6

The vector product is not commutative. The order in which the vectors are multiplied is important.

To account for order, remember:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

If $\vec{A}$ is parallel to $\vec{B}$ ($\theta = 0^{\circ}\ or 180^{\circ}$), then

$$\vec{A} \times \vec{B} = 0$$

Therefore

$$\vec{A} \times \vec{A} = 0$$

If $\vec{A}$ is perpendicular to $\vec{B}$, then

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|$$

6

Properties of the Vector Product

7

The vector product obeys the distributive law:

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

The derivative of the cross product with respect to some variable such as t is:

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \left(\frac{d\vec{A}}{dt} \times \vec{B}\right) + \left(\vec{A} \times \frac{d\vec{B}}{dt}\right)$$

where it is important to preserve the multiplicative order of the vectors.

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Vector Products of Unit Vectors

8

$$\hat{\imath} \times \hat{\imath} = (1)(1)sin0^{\circ} = 0$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

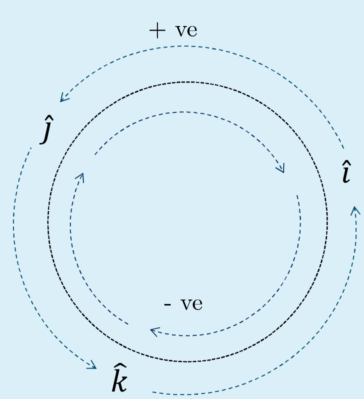
$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

$$\hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath}$$



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8

Signs in Cross Products

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- Signs are interchangeable in cross products

$$\vec{A} \times (-\vec{B}) = -\vec{A} \times \vec{B}$$

9

Using Determinants

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- The cross product can be expressed as

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = +(A_y B_z - A_z B_y)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

10

The Vector product-1

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [x] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Vectors $\vec{A}$ and $\vec{B}$ have magnitudes of 3 units and 4 units, respectively.

- What is the angle between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A} \times \vec{B} = 0$
- What is the angle between the directions of $\vec{A}$ and $\vec{B}$ if $\vec{A} \times \vec{B} = 12$

Solution:

$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta$$

$$0 = (3)(4)(\sin\theta)$$

$$\Rightarrow \ \theta = \sin^{-1} 0 = 0^\circ\ ,\ 180^\circ$$

The vectors are in the same direction (parallel); $\theta = 0^\circ$

OR: The vectors are in opposite directions (antiparallel); $\theta = 180^\circ$

Solution

$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta$$

$$12 = (3)(4)(\sin\theta)$$

$$\Rightarrow \ \theta = \sin^{-1} 1 = 90^\circ$$

The vectors are perpendicular; $\theta = 90^\circ$

The Vector product-2

Friday, 29 January, 2021 21:33

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

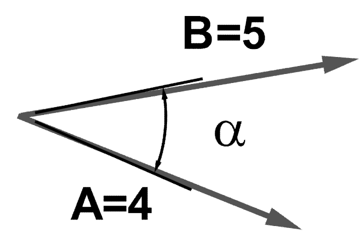
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

The figure shows two vectors lying in the xy plane, if $|\vec{A}| = 6$, $|\vec{B}| = 5$ and $\alpha = 40^{\circ}$. Determine the vector product $\vec{A} \times \vec{B}$ of them.



Answer:

$\vec{\mathrm{A}} \times \vec{B} = 19.3\hat{k}$

Cross product, unit-vector notation-1

Saturday, 30 January, 2021 12:20

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

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If $\vec{a} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{b} = -2\hat{\imath} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

SOLUTION

When two vectors are in unit-vector notation, we can find their cross product by using the distributive law.

$$\vec{c} = (3\hat{\imath} - 4\hat{\jmath}) \times \left(-2\hat{\imath} + 3\hat{k}\right) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -4 & 0 \\ -2 & 0 & 3 \end{vmatrix}$$

$$= +[(-4 \times 3) - (0 \times 0)]\hat{\imath} - [(3 \times 3) - (0 \times -2)]\hat{\jmath} + [(3 \times 0) - (-4 \times -2)]\hat{k}$$

$$\vec{c} = -12\hat{\imath} - 9\hat{\jmath} - 8\hat{k}$$

This vector is perpendicular to both $\vec{a}$ and $\vec{b}$, a fact you can check by showing that $\vec{a} \cdot \vec{c} = 0$ and $\vec{b} \cdot \vec{c} = 0$; that is, there is no component of $\vec{c}$ along the direction of either $\vec{a}$ or $\vec{b}$.