Chapter 3

1

VECTORS

1

Vectors

2

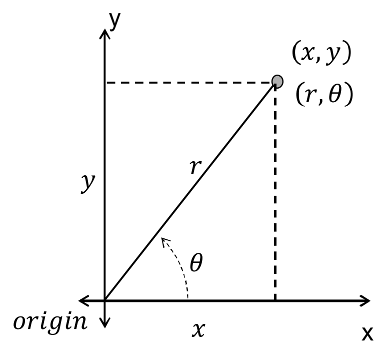
- Vector quantities
  - Physical quantities that have both numerical and directional properties.
- Mathematical operations of vectors in this chapter
  - Addition
  - Subtraction
  - Multiplication:
    - Multiplying with a scalar
    - Scalar product
    - Vector product

2

Coordinate Systems

3

- Used to describe the position of a point in space
- Common coordinate systems are:
  - Cartesian
  - Polar



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Polar to Cartesian Coordinates

4

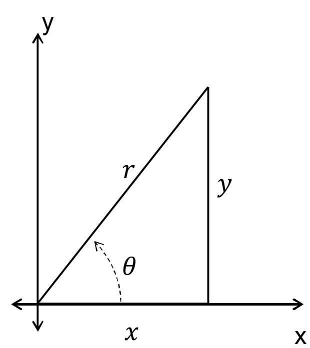
- Based on forming a right triangle from r and θ

$$cos\theta = \frac{x}{r} \Leftrightarrow x = rcos\theta$$

$$sin\theta = \frac{y}{r} \Leftrightarrow y = rsin\theta$$

- If the Cartesian coordinates are known:

$$r = \sqrt{x^2 + y^2}$$
$$tan\theta = \frac{y}{x} \Leftrightarrow \theta = tan^{-1}\frac{y}{x}$$



x: adjacent

y: opposite

r: hypotenuse

4

Vectors and Scalars

5

A scalar quantity is completely specified by a magnitude with an appropriate unit and has no direction.

- Many are always positive
- Some may be positive or negative
- Rules for ordinary arithmetic are used to manipulate scalar quantities.

A vector quantity is completely described by a magnitude, an appropriate units and a direction.

5

Vector and scalar Examples

6

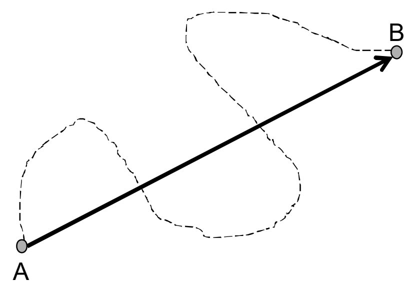
SCALARS	VECTORS
• Distance	• Displacement
• Speed	• Velocity
• Time intervals	• Acceleration
• Mass	• Force

6

Vector and scalar Example

7

- A particle travels from A to B along the path shown by the broken line.
  - This is the distance traveled and is a scalar.
- The displacement is the solid line from A to B
  - The displacement is independent of the path taken between the two points.
  - Displacement is a vector.



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Vector Notation

8

Bold font is used for printing: **A**

When dealing with just the magnitude of a vector in print, an italic letter will be used: A or $|\vec{A}|$

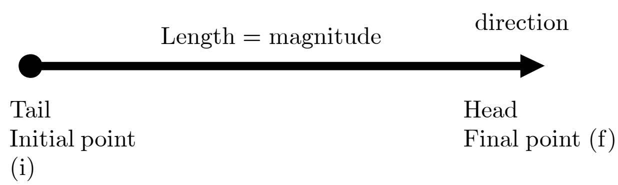
The magnitude of a vector is always a positive number.

When handwritten, use an arrow: $\vec{A}$

8

Vectors representation

9



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Vectors representation

10

$$\vec{A} = |\vec{A}|, \theta$$

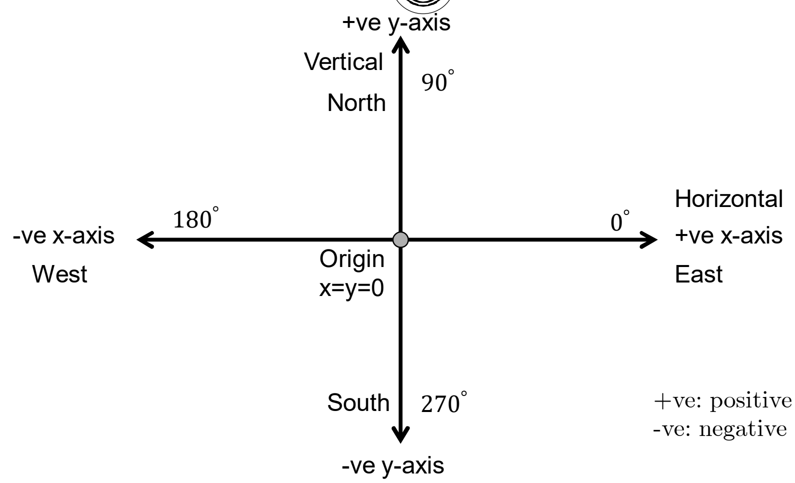
Vector A

Its magnitude

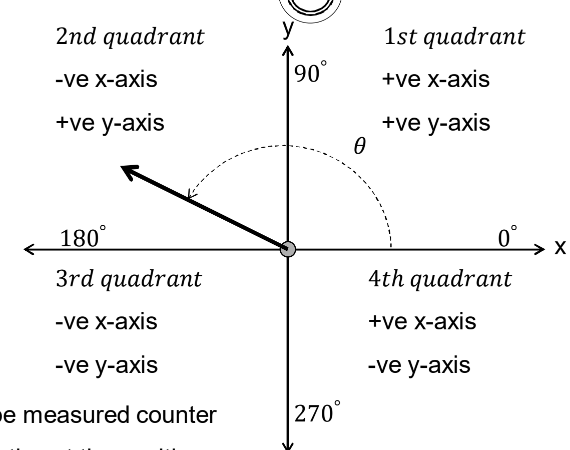
Its direction

10

Directions



Directions ($\theta \equiv Theta$)



(θ) : should be measured counter clockwise starting at the positive x-axis

Equality of Two Vectors

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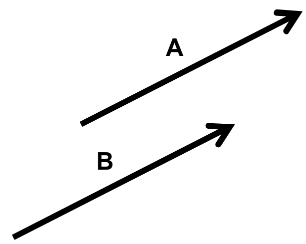
- Two vectors are equal if:
  - they have the same magnitude and
  - points in the same direction.



- $\vec{A} = \vec{B}$ if:
- $|\vec{A}| = |\vec{B}|$ and
  - In the same direction, or
  - Parallel, or
  - $\theta_{A,B} = 0^{\circ}$

13

Adding Vectors

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- Vector addition is very different from adding scalar quantities.
- When adding vectors, their directions must be taken into account.
- Units must be the same
- Graphical methods
  - Use scale drawings
- Algebraic methods
  - More convenient

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Adding Vectors Graphically

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- Choose a scale.
- Draw the first vector, $\vec{A}$, with the appropriate length and in the direction specified, with respect to a coordinate system.
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$.
- Continue drawing the vectors "tip-to-tail" or "head-to-tail".
- The resultant is drawn from the origin of the first vector to the end of the last vector.

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Adding Vectors Graphically, cont.

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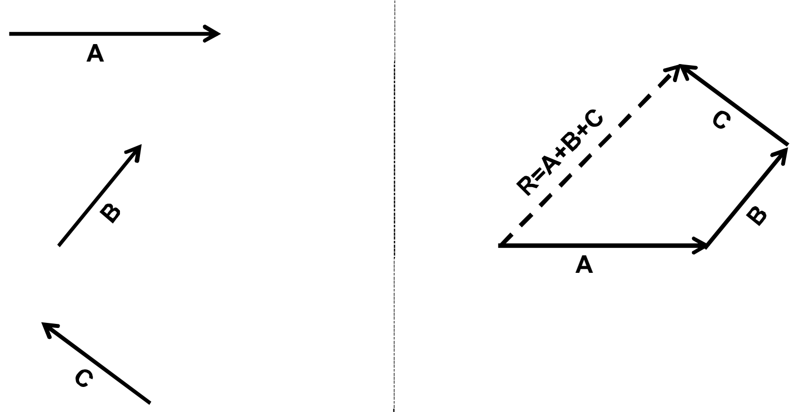

R: resultant

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Adding Vectors, Rules

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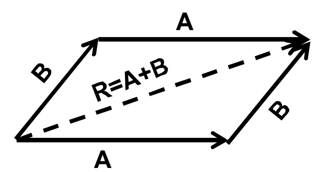
- The Commutative Law of Addition: when two vectors are added, the sum is independent of the order of the addition.
  - This is

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$



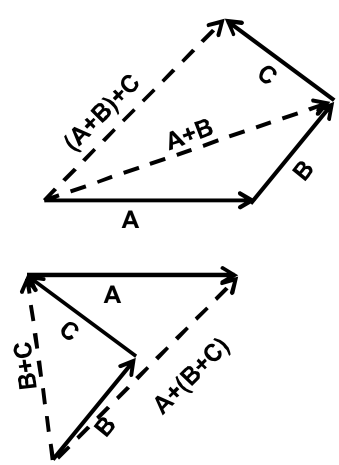
17

Adding Vectors, Rules cont.

18

- The Associative Property of Addition: when adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped.

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$



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Adding Vectors, Rules final

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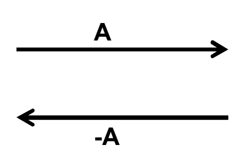
- When adding vectors, all of the vectors must have the same units.
- All of the vectors must be of the same type of quantity.
  - For example, you cannot add a displacement to a velocity.

Negative of a Vector

20

- The negative of a vector is defined as the vector that, when added to the original vector, gives a resultant of zero.
  - Represented as $-\vec{A}$
  - $\vec{A} + (-\vec{A}) = 0$
- The negative of the vector will have the same magnitude, but points in the opposite direction.



Subtracting Vectors

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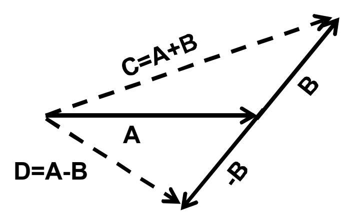
- Special case of vector addition.
- If $\vec{A} - \vec{B}$, then use $\vec{A} + (-\vec{B})$
- Continue with standard vector addition procedure.



21

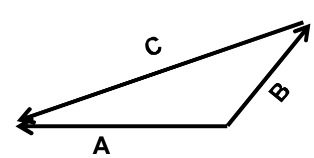
Subtracting Vectors, Method 2

22

In the figure shown:

$$\vec{B} + \vec{C} = \vec{A}$$
$$\vec{C} = \vec{A} - \vec{B}$$

- The resultant vector $\vec{C}$ of subtracting $\vec{B}$ from $\vec{A}$ points from the tip of the second to the tip of the first.



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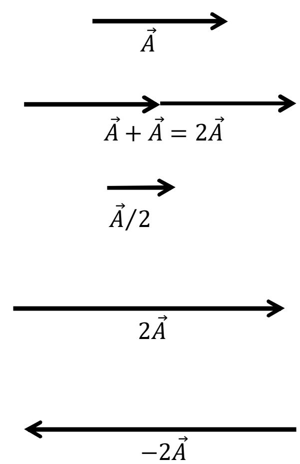
Multiplying or Dividing a Vector by a Scalar

23

- The result of the multiplication or division of a vector by a scalar is a vector.
- The magnitude of the vector is multiplied or divided by the scalar.
- If the scalar is positive, the direction of the result is the same as of the original vector.
- If the scalar is negative, the direction of the result is opposite that of the original vector.



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Component Method of Adding Vectors

24

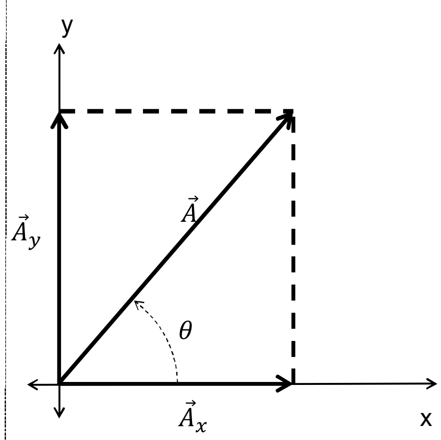
- Graphical addition is not recommended when:
  - High accuracy is required
  - If you have a three-dimensional problem
- Component method is an alternative method
  - It uses projections of vectors along coordinate axes

24

Components of a Vector, Introduction

25

- A **component** is a projection of a vector along an axis.
  - A_x : the component (projection) of the vector along the x-axis.
  - A_y : the component (projection) of the vector along the y-axis.



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Components of a Vector, Introduction

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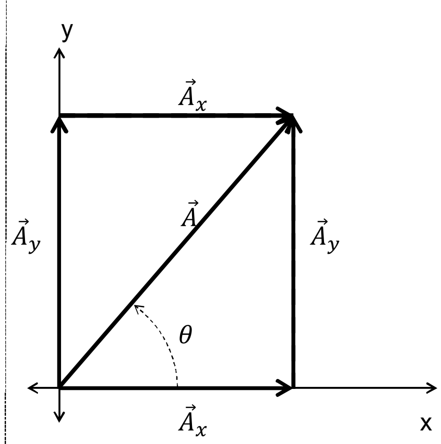
$$\vec{A}_x + \vec{A}_y = \vec{A}$$

$$\vec{A}_y + \vec{A}_x = \vec{A}$$



26

Vector Component Terminology

27

- $\vec{A}_x$ and $\vec{A}_y$ are the component vectors of $\vec{A}$.
  - They are vectors and follow all the rules for vectors.
- $|\vec{A}_x|$ and $|\vec{A}_y|$ are scalars, and will be referred to as the components of $\vec{A}$.

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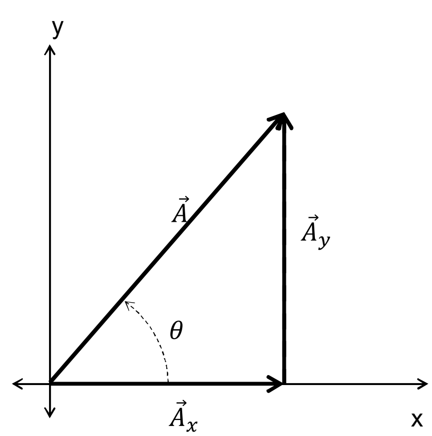
Components of a Vector

28

- Assume you are given a vector $\vec{A}$
- It can be expressed in terms of two other vectors, $\vec{A}_x$ and $\vec{A}_y$

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

- These three vectors form a right triangle.



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Components of a Vector

29

The x-component is:

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{A_x}{A}$$

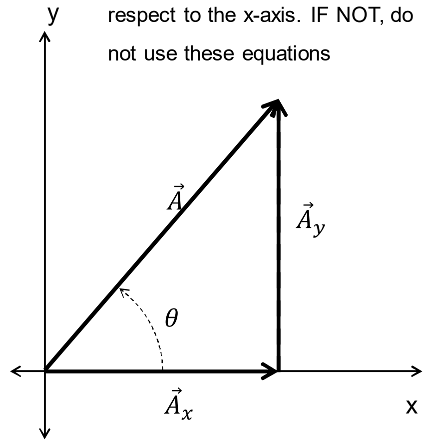
$$A_x = A\cos\theta$$

The y-component is:

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{A_y}{A}$$

$$A_y = A\sin\theta$$

The angle θ is measured with respect to the x-axis. IF NOT, do not use these equations



29

Components of a Vector

30

The magnitude of the vector is:

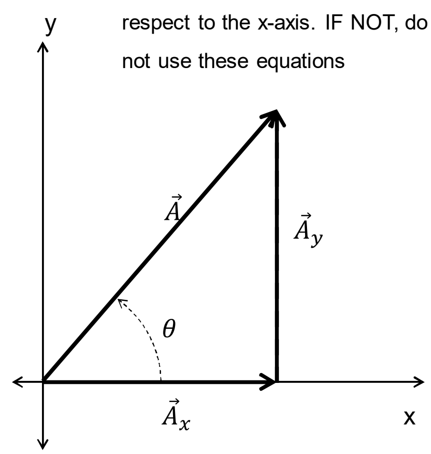
$$A = \sqrt{A_x^2 + A_y^2}$$

The direction of the vector is:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

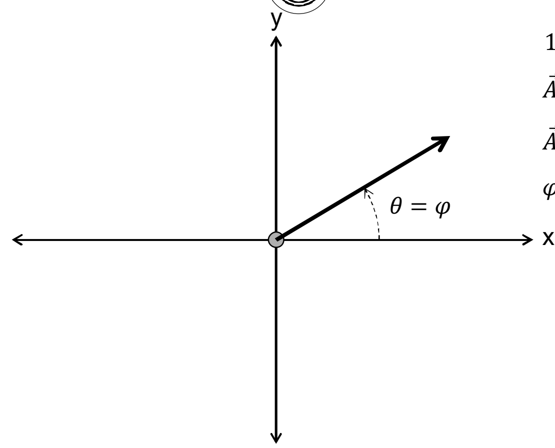
The angle θ is measured with respect to the x-axis. IF NOT, do not use these equations



30

Direction (θ) and calculations from your calculator (φ)

31



$1st\ quadrant$

$\vec{A}_x$: +ve

$\vec{A}_y$: +ve

φ : +ve

31

Direction (θ) and calculations from your calculator (φ)

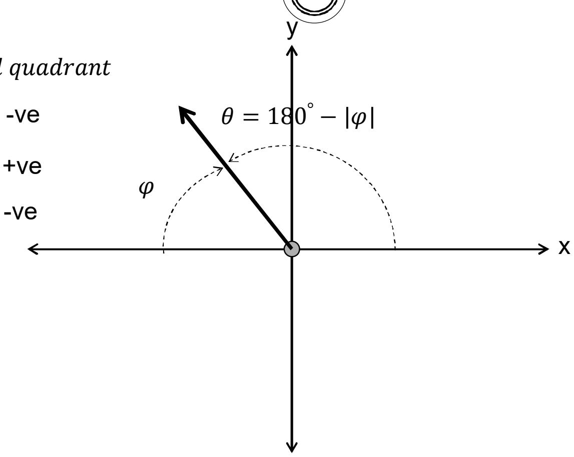
32

$2nd\ quadrant$

$\vec{A}_x$: -ve

$\vec{A}_y$: +ve

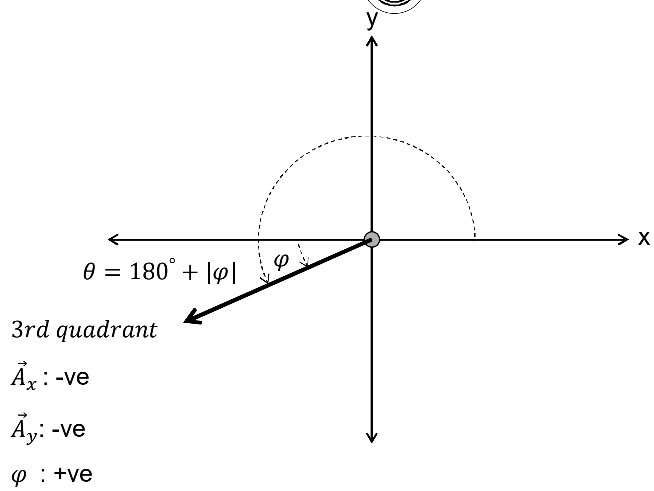
φ : -ve



32

Direction (θ) and calculations from your calculator (φ)

33



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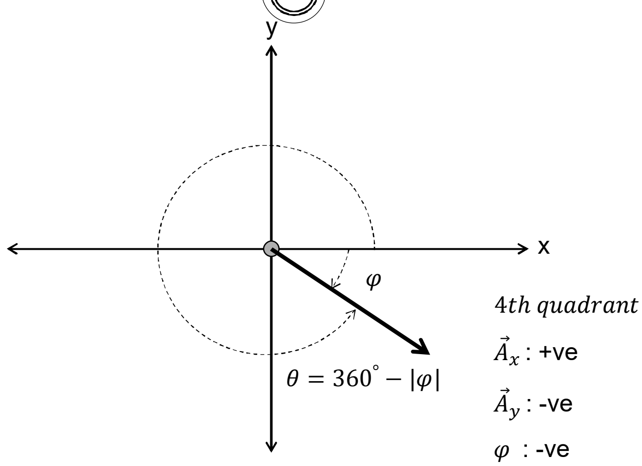
Direction (θ) and calculations from your calculator (φ)

34



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Unit Vectors

35

- A unit vector is a dimensionless vector with a magnitude of exactly 1.
- Unit vectors are used to specify a direction and have no other physical significance.

Unit Vectors, cont.

36

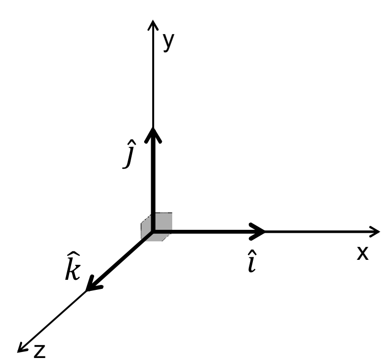
- The symbols $\hat{\imath}, \hat{\jmath}$ *and* $\hat{k}$ represent unit vectors.
- The magnitude of each unit vector is 1.

$$\hat{\imath} = \hat{\jmath} = \hat{k} = 1$$

- They form a set of mutually perpendicular vectors in a right-handed coordinate system.

$$\hat{\imath} \perp \hat{\jmath} \perp \hat{k}$$

- Dimensionless vectors.



Unit Vectors in Vector Notation

37

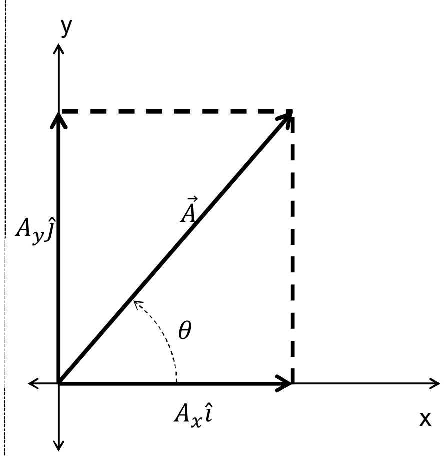
- $\vec{A}_x$ is the same as $A_x\hat{\imath}$, $\vec{A}_y$ is the same as $A_y\hat{\jmath}$ and $\vec{A}_z$ is the same as $A_z\hat{k}$.
- The complete vector can be expressed as:

$$\vec{A} = |\vec{A}|, \theta$$

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{A} = Acos\theta\hat{\imath} + Asin\theta\hat{\jmath}$$



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Position Vector, Example

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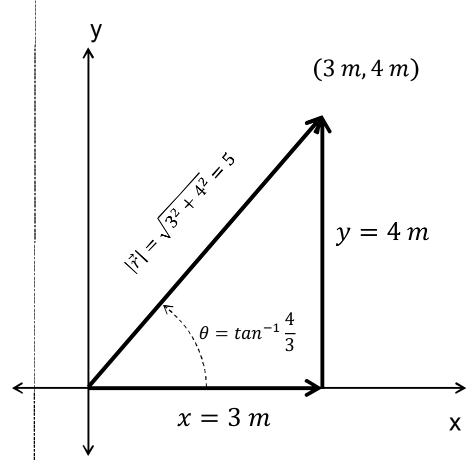
- If $x = 3m$ and $y = 4m$:

$\vec{r} = 3 + 4 = 7$ X

$\vec{r} = \vec{3}_x + \vec{4}_y$?

$\vec{r} = 3\hat{\imath} + 4\hat{\jmath}$ √



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Adding Vectors Using Unit Vectors

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- Given:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

- Find: $\vec{R} = \vec{A}+\vec{B}$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath}$$

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Adding Vectors Using Unit Vectors

40

So, the x-component of the resultant is:

$$R_x = A_x + B_x$$

The y-component of the resultant is:

$$R_y = A_y + B_y$$

The magnitude of the resultant is:

$$R = \sqrt{R_x^2 + R_y^2}$$

The direction of the resultant is:

$$\theta = tan^{-1}\frac{R_y}{R_x}$$

40

Three-Dimensional Extension

41

- Given:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

- Find: $\vec{R} = \vec{A}+\vec{B}$

$$\vec{R} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

$$\vec{R} = R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}$$

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Three-Dimensional Extension

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So, the x-component of the resultant is:

$$R_x = A_x + B_x$$

The y-component of the resultant is:

$$R_y = A_y + B_y$$

The z-component of the resultant is:

$$R_z = A_z + B_z$$

The magnitude of the resultant is:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

The direction of the resultant is:

LATER

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Adding Three or More Vectors

43

- The same method can be extended to adding three or more vectors.
- Assume

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k}$$

- And

$$\vec{R} = \vec{A}+\vec{B}+\vec{C}$$

$$\vec{R} = (A_x + B_x + C_x)\hat{\imath} + \left(A_y + B_y + C_y\right)\hat{\jmath} + (A_z + B_z + C_z)\hat{k}$$

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Scalar Product of Two Vectors

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The scalar product of two vectors is defined as.

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \operatorname{Cos}\theta$$

- It is also called the dot product.

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Scalar Product of Two Vectors

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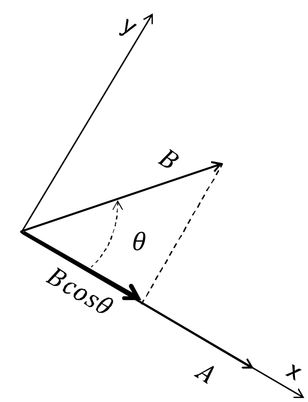
- θ is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn starting at the same point.
- The scalar product of any two vectors is a scalar quantity.



45

Scalar Product, cont

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The scalar product is commutative.

$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$

The scalar product obeys the distributive law of multiplication.

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\left(\vec{A}\cdot\vec{B}\right)+\left(\vec{A}\cdot\vec{C}\right)$$

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Dot Products of Unit Vectors

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$$\hat{\imath} \cdot \hat{\imath} = (1)(1)cos0^{\circ} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = (1)(1)cos90^{\circ} = 0$$

- In general:

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = 0$$

- Using component form with vectors:

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$\Rightarrow \quad \vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Dot Products of Unit Vectors

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Special cases:

$$\vec{A} \cdot \vec{B} = \begin{cases} AB; If\ they\ are\ parallel\ ;\ \theta = 0^{\circ} \\ -AB\ ;\ If\ they\ are\ anti-parallel\ ;\ \theta = 180^{\circ} \\ 0\ ;If\ they\ are\ perpendicular\ ;\ \theta = 90^{\circ} \end{cases}$$

$$\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

The Vector Product

49

- There are instances where the product of two vectors is another vector.
  - Earlier we saw where the product of two vectors was a scalar.
- The vector product of two vectors is called the cross product.

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The Vector Product Defined

50

The vector (cross) product of $\vec{A}$ and $\vec{B}$ is defined as a third vector:

$$\vec{C} = \vec{A} \times \vec{B}$$

The magnitude of $\vec{C}$:

$$|\vec{C}| = |\vec{A}||\vec{B}|sin\theta$$

θ is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn starting at the same point.

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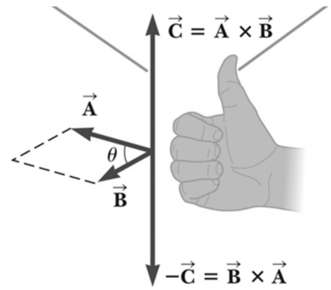
More About the Vector Product

51

The direction of $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.

The best way to determine this direction is to use the right-hand rule (R.H.R).



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Properties of the Vector Product

52

The vector product is not commutative. The order in which the vectors are multiplied is important.

To account for order, remember:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

If $\vec{A}$ is parallel to $\vec{B}$ ($\theta = 0^{\circ}\ or 180^{\circ}$), then

$$\vec{A} \times \vec{B} = 0$$

Therefore

$$\vec{A} \times \vec{A} = 0$$

If $\vec{A}$ is perpendicular to $\vec{B}$, then

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|$$

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Properties of the Vector Product

53

The vector product obeys the distributive law:

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

The derivative of the cross product with respect to some variable such as t is:

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \left(\frac{d\vec{A}}{dt} \times \vec{B}\right) + \left(\vec{A} \times \frac{d\vec{B}}{dt}\right)$$

where it is important to preserve the multiplicative order of the vectors.

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Vector Products of Unit Vectors

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$$\hat{\imath} \times \hat{\imath} = (1)(1)sin0^{\circ} = 0$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$

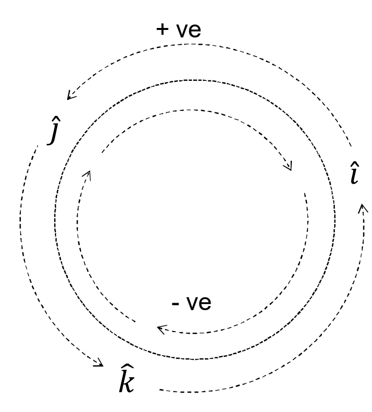
$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

$$\hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath}$$



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Signs in Cross Products

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- Signs are interchangeable in cross products

$$\vec{A} \times (-\vec{B}) = -\vec{A} \times \vec{B}$$

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Using Determinants

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- The cross product can be expressed as

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = +(A_y B_z - A_z B_y)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

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EXAMPLE (1)

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The magnitudes of the vectors $\vec{A}$ and $\vec{B}$ are (3) and (4), respectively, and $\vec{C} = \vec{A} + \vec{B}$

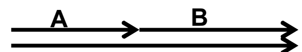
- (A) What is the maximum possible magnitude for $\vec{C}$?
- (B) What is the minimum possible magnitude for $\vec{C}$?

Solution

(A) has a maximum possible magnitude when both vectors are in same direction:

$$\vec{C} = \vec{A} + \vec{B}$$

$$|\vec{C}| = |\vec{A}| + |\vec{B}| = 3 + 4 = 7$$



(B) has a minimum possible magnitude when both vectors are in opposite directions:

$$\vec{C} = \vec{A} + \vec{B}$$

$$|\vec{C}| = |\vec{B}| - |\vec{A}| = 4 - 3 = 1$$

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EXAMPLE (2)

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A car travels (20 km) due north and then (35 km) in a direction (60°) west of north as shown.

a) Write the two displacements in magnitude – direction notation.
b) Determine the x – and y – components of the two displacements.
c) Write the two displacements in unit vector notation.
d) Find the resultant displacement in unit vector notation.
e) Determine the x – and y – components of the resultant displacement.
f) Find the magnitude of the resultant displacement.
g) Find the direction of the resultant displacement.
h) Write the resultant displacement in magnitude – direction notation.

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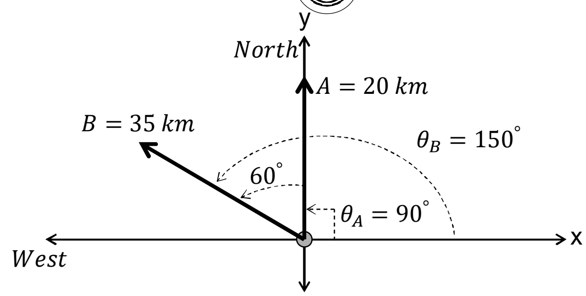
Solution

a)



$\vec{A} = 20\ km, 90^\circ$ $\vec{B} = 35\ km, 150^\circ$

Solution

b) $A_x = 20\cos 90^\circ = 0\ km$

$A_y = 20\sin 90^\circ = 20\ km$

$B_x = 35\cos 150^\circ = -30.3\ km$

$B_y = 35\sin 150^\circ = 17.5\ km$

c) $\vec{A} = (0\,\hat{\imath} + 20\,\hat{\jmath})\ km$

$\vec{B} = (-30.3\,\hat{\imath} + 17.5\,\hat{\jmath})\ km$

d) $\vec{C} = \vec{A} + \vec{B} = [(0 + {}^{-}30.3)\hat{\imath} + (20 + 17.5)\hat{\jmath}]km$

$\Rightarrow\ \vec{C} = ({}^{-}30.3\hat{\imath} + 37.5\,\hat{\jmath})\ km$

e) $C_x = {}^{-}30.3\ km$

$C_x = 37.5\ km$

Solution

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f) $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(^-30.3)^2 + (37.5)^2} = 48.2\ km$

g) From your calculator:

$$\varphi = tan^{-1}\frac{C_y}{C_x} = tan^{-1}\frac{37.5}{^-30.3} \cong -51^{\circ}$$

Since C_x is $-ve$ and C_y is $+ve$, then the resultant displacement lies in the 2^{nd} quadrant, and its direction is:

$$\theta = 180^{\circ} - |\varphi| = 180^{\circ} - 51^{\circ} = 129^{\circ}$$

h) $\vec{C} = 48.2\ km, 129^{\circ}$

Example (3)

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Find the sum of three displacement vectors lying in the xy plane and given by: $\vec{A} = (4.2\,\hat{\imath} - 1.5\,\hat{\jmath})\ m$, $\vec{B} = (-1.6\,\hat{\imath} + 2.9\,\hat{\jmath})\ m$ and $\vec{C} = (-3.7\,\hat{\jmath})\ m$.

Example (4)

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- A particle undergoes three consecutive displacements: $\vec{A} = (15\,\hat{\imath} + 30\,\hat{\jmath} + 12\,\hat{k})\,m$, $\vec{B} = (23\,\hat{\imath} - 14\,\hat{\jmath} - 5\,\hat{k})\,m$ and $\vec{C} = (-13\,\hat{\imath} + 15\,\hat{\jmath})\,m$. Find unit-vector notation for the resultant displacement and its magnitude.

Example (5)

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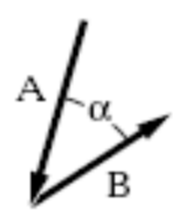
- The figure shows two vectors lying in the xy plane, if (A = 6), (B = 5), and (α = 40°). Determine the scalar product of them.



Example (6)

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- Vectors **A** and **B** have magnitudes of 3 units and 4 units, respectively.
- a) What is the angle between the directions of **A** and **B** if **A.B**=0 .
- b) What is the angle between the directions of **A** and **B** if **A.B**=12 .
- c) What is the angle between the directions of **A** and **B** if **A.B**= -12 .

Example (7)

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- The vectors **A** and **B** are given by: $\vec{A} = (3\,\hat{\imath} - 4\,\hat{\jmath})$ and $\vec{B} = \left(-2\,\hat{\imath} + 3\,\hat{k}\right)$.
- a) Find the magnitude of the two vectors.
- b) Determine the scalar product of the two vectors.
- c) Find the angle between the directions of the two vectors.

Example (8)

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- Find the angle between $\vec{A} = (3\,\hat{\imath} - \hat{\jmath} + 5\,\hat{k})$ and the positive x-axis.

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Vector Product Example

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- Given $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}};\ \vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$
- Find $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$
- Result

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$
$$= 2\hat{\mathbf{i}} \times (-\hat{\mathbf{i}}) + 2\hat{\mathbf{i}} \times 2\hat{\mathbf{j}} + 3\hat{\mathbf{j}} \times (-\hat{\mathbf{i}}) + 3\hat{\mathbf{j}} \times 2\hat{\mathbf{j}}$$
$$= 0 + 4\hat{\mathbf{k}} + 3\hat{\mathbf{k}} + 0 = 7\hat{\mathbf{k}}$$

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Torque Vector Example

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- Given the force and location

$$\vec{\mathbf{F}} = (2.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})\,N$$

$$\vec{\mathbf{r}} = (4.00\,\hat{\mathbf{i}} + 5.00\,\hat{\mathbf{j}})\,m$$

- Find the torque produced

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = [(4.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\mathrm{N}] \times [(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})\mathrm{m}]$$

$$= [(4.00)(2.00)\hat{\mathbf{i}} \times \hat{\mathbf{i}} + (4.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$

$$+(5.00)(2.00)\hat{\mathbf{j}} \times \hat{\mathbf{i}} + (5.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$

$$= 2.0\,\hat{\mathbf{k}}\ \mathrm{N \cdot m}$$

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