Chapter 4

1

Motion in two dimensions

Position and Displacement

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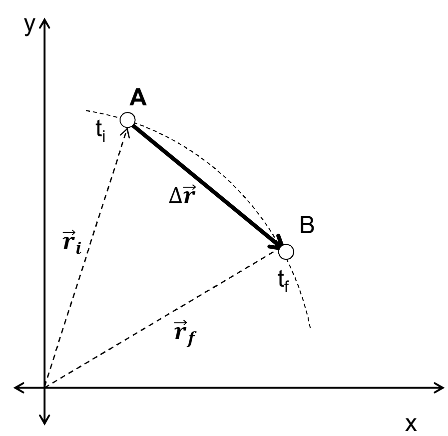
The position of an object is described by its position vector, $\vec{r}$, with respect to a chosen reference point (the origin).

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

The displacement of the object is the *change in its position.*

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



Velocity and Acceleration

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Average Velocity:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

Instantaneous Velocity:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Average Acceleration:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Kinematic Equations for 2-D Motion

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- These equations will be similar to those of one-dimensional kinematics.
- Motion in two dimensions can be modeled as two independent motions in each of the two perpendicular directions associated with the x and y axes.
  - Any influence in the y direction does not affect the motion in the x direction.

Kinematic Equations

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Since acceleration is constant, we can also find an expression for the velocity as a function of time:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

The position vector can also be expressed as a function of time:

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

Kinematic Equations: 2-D

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Equations	Missing
$\vec{v}_f = \vec{v}_i + \vec{a}t$	$\Delta\vec{r}$: displacement (m)
$\Delta\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	$\vec{v}_f$: final velocity (m/s)
$\Delta\vec{r} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$	$\vec{v}_i$: initial velocity (m/s)
$\Delta\vec{r} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)t$	$\vec{a}$: acceleration (m/s^2)

Kinematic Equations: Horizontal components

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Equations	Missing
$v_{fx} = v_{ix} + a_x t$	Δx : displacement (m)
$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$	t : time (s)
$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$	v_{fx} : final velocity (m/s)
$\Delta x = v_{fx} t - \frac{1}{2} a_x t^2$	v_{ix} : initial velocity (m/s)
$\Delta x = \frac{1}{2}(v_{ix} + v_{fx}) t$	a_x: acceleration (m/s^2)

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Kinematic Equations: Vertical components

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Equations	Missing
$v_{fy} = v_{iy} + a_y t$	Δy : displacement (m)
$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$	t : time (s)
$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$	v_{fy} : final velocity (m/s)
$\Delta y = v_{fy} t - \frac{1}{2} a_y t^2$	v_{iy} : initial velocity (m/s)
$\Delta y = \frac{1}{2}(v_{iy} + v_{fy}) t$	a_y: acceleration (m/s^2)

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Projectile Motion

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An object may move in both the x and y directions simultaneously.

The form of two-dimensional motion we will deal with is called projectile motion.

Assumptions of Projectile Motion

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The free-fall acceleration is constant over the range of motion.

- It is directed downward.

The effect of air friction is negligible.

With these assumptions, an object in projectile motion will follow a parabolic path.

- This path is called the trajectory.

Projectile Motion

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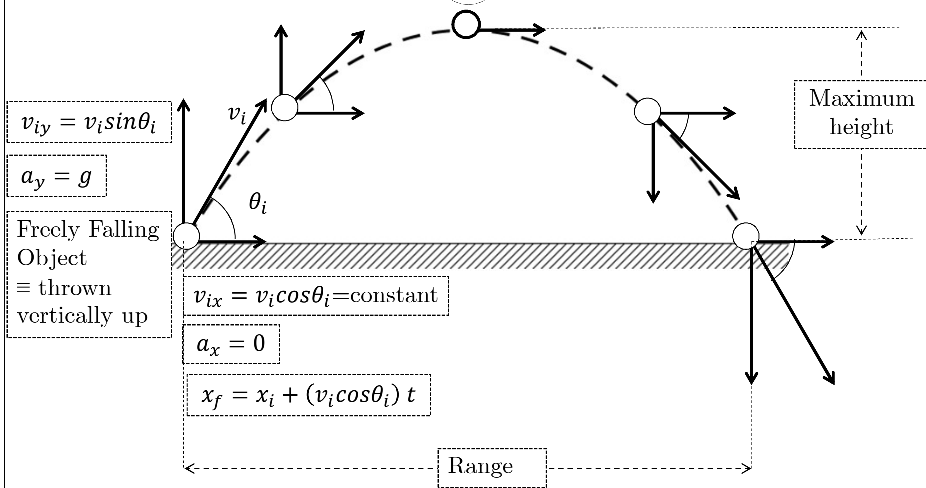

Thrown up from height H

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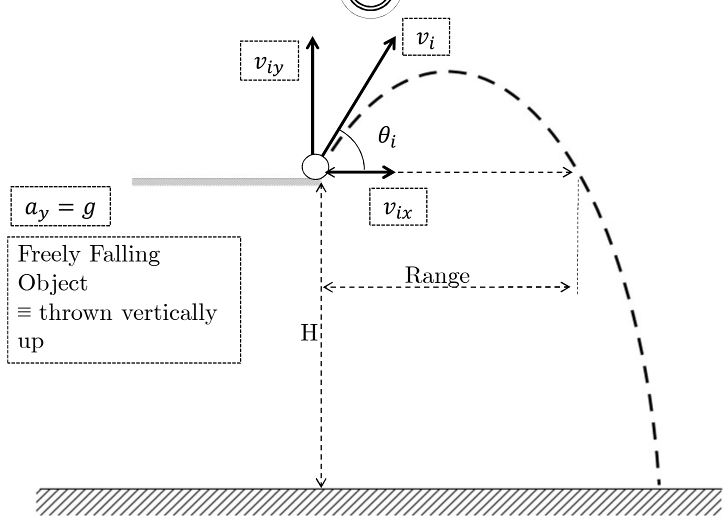

Thrown down from height H

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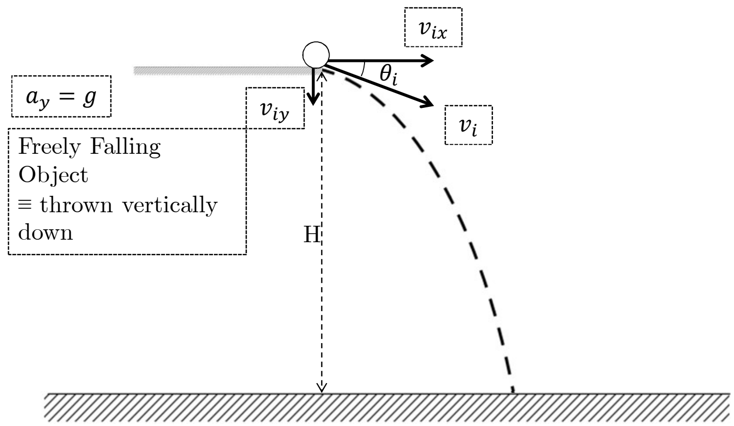

Thrown horizontally from height H

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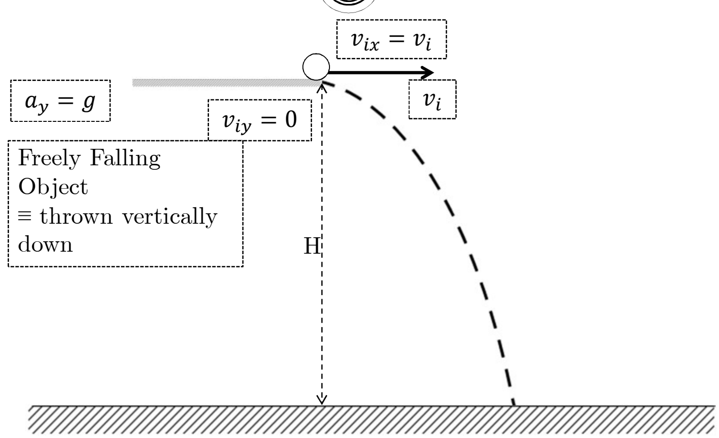

Kinematic Equations

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$$\vec{a} = \vec{g} = -9.8\hat{j}(m/s^2)$$

Equations	Missing
$\vec{v}_f = \vec{v}_i + \vec{a}t$	$\Delta\vec{r}$: displacement (m)
$\Delta\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$	$\vec{v}_f$: final velocity (m/s)
$\Delta\vec{r} = \vec{v}_f t - \frac{1}{2}\vec{a}t^2$	$\vec{v}_i$: initial velocity (m/s)

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Kinematic Equations: Vertical components

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$$g = 9.8\ m/s^2$$

Equations	Missing
$v_{fy} = (v_i sin\theta_i) - gt$	Δy : displacement (m)
$v_{fy}^2 = (v_i sin\theta_i)^2 - 2g\Delta y$	t : time (s)
$\Delta y = (v_i sin\theta_i)t - \frac{1}{2}gt^2$	v_f : final velocity (m/s)
$\Delta y = v_{fy}t + \frac{1}{2}gt^2$	v_i : initial velocity (m/s)

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Kinematic Equations:

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Horizontal components: it is a motion with constant velocity:

- $x_f = x_i + (v_i cos\theta_i)\ t$

Range: is the maximum horizontal distance:

- $R = \frac{v_i^2 sin2\theta_i}{g}$

Maximum height: is the maximum vertical distance:

- $h = \frac{v_i^2 sin^2\theta_i}{2g}$

Notes

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The horizontal and vertical components of a projectile's motion are completely independent of each other.

Time is the common variable for both components:

- $t_x = t_y = t$

The velocity at the maximum height equals the horizontal component:

- $v_{max.h} = v_{ix} = v_i cos\theta$

The acceleration anywhere along the trajectory is:

- $\vec{a} = \vec{g} = -9.8\hat{j}(m/s^2)$

The path of a projectile is a parabola.

Notes, …

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The maximum range occurs at:

- $\theta_i = 45^\circ$.

Complementary angles will produce the same range.

- The maximum height will be different for the two angles.
- The times of the flight will be different for the two angles.

Uniform Circular Motion

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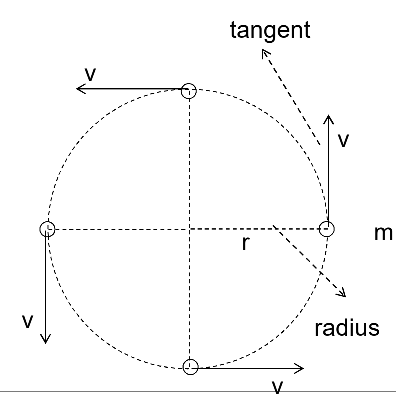
Uniform circular motion occurs when an object moves in a circular path with a constant speed.

The constant-magnitude velocity vector is always tangent to the path of the object.

An acceleration exists since the direction of the motion is changing .



Changing Velocity in Uniform Circular Motion

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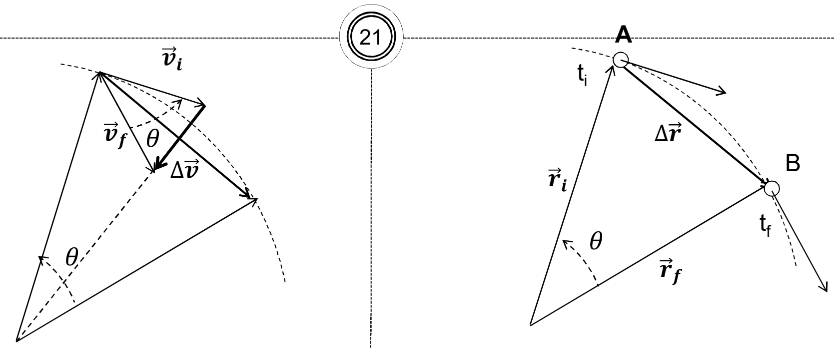

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Changing Velocity in Uniform Circular Motion

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The change in the velocity vector is due to the change in direction.

The direction of the change in velocity is toward the center of the circle.

The vector diagram shows:

- $\vec{v}_f = \vec{v}_i + \Delta\vec{v}$

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Centripetal Acceleration

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The acceleration is always perpendicular to the path of the motion.

The acceleration always points toward the center of the circle of motion.

This acceleration is called the centripetal acceleration.

- It is also called the radial acceleration.

Centripetal Acceleration, cont

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The magnitude of the centripetal acceleration vector is given by

- $a_c = \frac{v^2}{r}$

The direction of the centripetal acceleration vector is always changing, to stay directed toward the center of the circle of motion.

Period

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The period, T, is the time required for one complete revolution.

The speed of the particle would be the circumference of the circle of motion divided by the period.

- $v = \frac{2\pi r}{T}$

Therefore, the period is defined as

- $T = \frac{2\pi}{v}$

Tangential Acceleration

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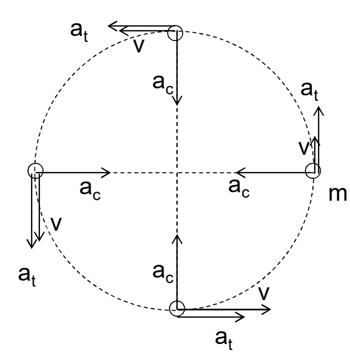
The magnitude of the velocity could also be changing.

In this case, there would be a tangential acceleration.

The motion would be under the influence of both tangential and centripetal accelerations.



Tangential Acceleration, equation

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The tangential acceleration:

- $a_t = \left|\frac{dv}{dt}\right|$
- Direction: Same as velocity vector if (v) is increasing, opposite if (v) is decreasing

Total Acceleration

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The tangential acceleration causes the change in the speed of the particle.

The centripetal acceleration comes from a change in the direction of the velocity vector.

The total acceleration:

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

Its magnitude:

$$a = \sqrt{a_c^2 + a_t^2}$$