Chapter 6

1

Centripetal Force

1

Uniform Circular Motion, Force

2

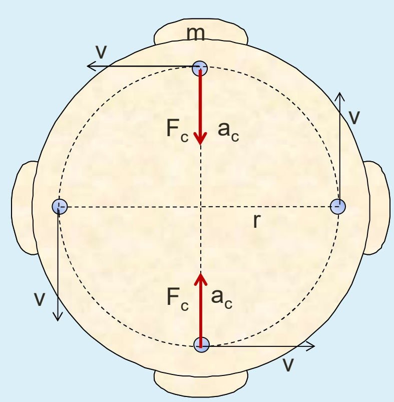
According to Newton's second law, all accelerations are caused by a net force.

In circular motion, the net force is called the centripetal force:

$$\sum F = ma_c$$

$$F_c = \frac{mv^2}{r}$$

The force is also directed toward the center of the circle.



2

Uniform Circular Motion, Force

3

The centripetal force is NOT a new kind of force;

It is any force that keeps an object moving in a circular path without changing its speed.

It causes a change in the direction of the velocity.

It can be a tension force, a frictional force, a gravitational force, … ,.

Without this force, an object will simply continue moving in straight line.

How Fast Can It Spin?

Friday, 29 January, 2021 21:35

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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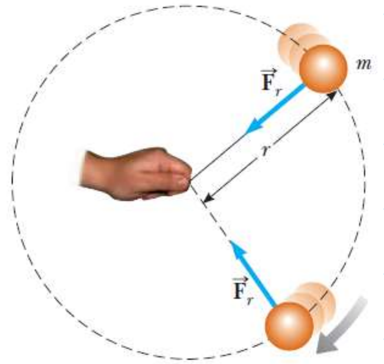
A puck of mass 0.5 kg is attached to the end of a cord 1.5 m long. The puck moves in a horizontal circle as shown. If the cord can withstand a maximum tension of 50 N, what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.



Solution

Because the force causing the centripetal acceleration in this case is the force T exerted by the cord on the ball:

$$\sum F_c = ma_c \Rightarrow T = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rT}{m}}$$

The Conical Pendulum

Saturday, 30 January, 2021 13:26

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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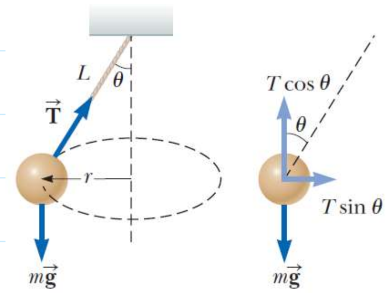
A small ball of mass (m) is suspended from a string of length (L). The ball revolves with constant speed (v) in a horizontal circle of radius (r) as shown. Find an expression for v.



Solution

The ball does not accelerate vertically. Therefore, we model it as a particle in equilibrium in the vertical direction. It experiences a centripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction.

The force $\vec{T}$ exerted by the string on the ball is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of the circular path.

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = 0 \Rightarrow T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg \ldots\ldots(1)$$

Apply the particle in uniform circular motion model in the horizontal direction:

$$\sum F_x = ma_c \Rightarrow T\sin\theta = \frac{mv^2}{r} \ldots\ldots(2)$$

deviding eq.(2) by eq.(1):

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta} \ldots\ldots(3)$$

$$But:\ \sin\theta = \frac{r}{L} \Rightarrow r = L\sin\theta \ldots\ldots(4)$$

substituting eq.(4) into eq.(3): $\Rightarrow \boxed{v = \sqrt{Lg\sin\theta\tan\theta}}$

What Is the Maximum Speed of the Car?

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A car of mass m moving on a flat, horizontal road negotiates a curve of radius r as shown. If the coefficient of static friction between the tires and dry pavement is μ_s, find the maximum speed the car can have and still make the turn successfully.

SOLUTION

Imagine that the curved roadway is part of a large circle so that the car is moving in a circular path.

The force that enables the car to remain in its circular path is the force of static friction. (It is *static* because no slipping occurs at the point of contact between road and tires. If this force of static friction were zero – for example, if the car were on an icy road – the car would continue in a straight line and slide off the curved road.) The maximum speed *v*max the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f\hat{s}_{s,max}$.

Apply the particle in uniform circular motion model in the horizontal direction:

$$\sum F_x = ma_c \;\Rightarrow\; fs\frac{mv_{max}^2}{r\,(1)}_{s,max}$$

Apply the particle in equilibrium model in the vertical direction:

$$\sum F_y = 0 \;\Rightarrow\; n - mg = 0 \;\Rightarrow\; n = mg\,.....(2)$$

Solve Equation (1) for the maximum speed and substitute for *n*:

$$v\sqrt{\mu_s r g}_{max}$$

The Banked Roadway

Saturday, 30 January, 2021 13:27

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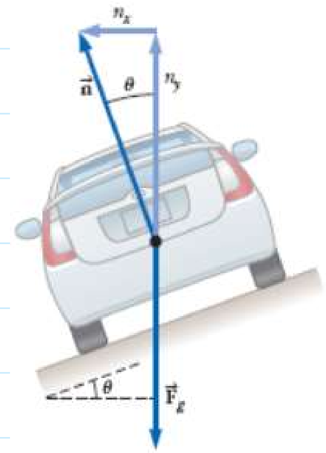
A civil engineer wishes to redesign the horizontal curved roadway in such a way that a car will not have to rely on friction to round the curve without skidding. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. At what angle should the curve be banked?



SOLUTION

The normal force $\hat{n}$ has a horizontal component toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $\hat{n}_x = n\sin\theta$ causes the centripetal acceleration.

Write Newton's second law for the car in the radial direction, which is the x direction:

$$\sum F_x = ma_c \Rightarrow n\sin\theta = \frac{mv^2}{r} \;.....(1)$$

Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = 0 \Rightarrow n\cos\theta - mg = 0 \Rightarrow n\cos\theta = mg \;.....(2)$$

Divide Equation (1) by Equation (2):

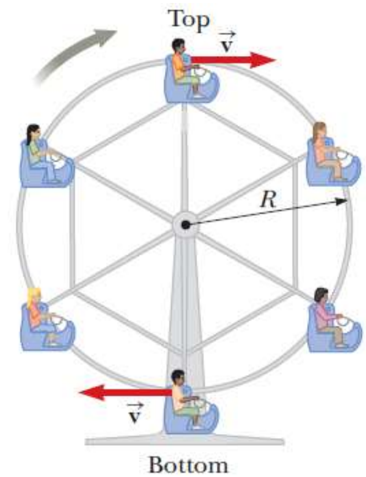
$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

Riding the Ferris Wheel

Saturday, 30 January, 2021 13:29

A child of mass (m) rides on a Ferris wheel as shown. The child moves in a vertical circle of radius (r) at a constant speed (v).

- Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg.
- Determine the force exerted by the seat on the child at the top of the ride.



Solution

At both the bottom of the path and the top, the only forces acting on him are the downward gravitational force ($m\vec{g}$) and the upward force ($\hat{n}_{bot}$) exerted by the seat and in opposite directions. The vector sum of these two forces gives a force of constant magnitude ($n_{bot} - mg$) that keeps the child moving in a circular path at a constant speed.

Apply Newton's second law to the child in the radial direction:

$$\sum F_r = ma_c \Rightarrow n_{bot} - mg = \frac{mv^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{bot} = mg + \frac{mv^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

Hence, the magnitude of the force exerted by the seat on the child is greater than the weight of the child.

Solution

The net downward force that provides the centripetal acceleration has a magnitude ($mg - n_{top}$).

Apply Newton's second law to the child in the radial direction:

$$\sum F_r = ma_c \Rightarrow mg - n_{top} = \frac{mv^2}{r}$$

Solve for the force exerted by the seat on the child:

$$n_{top} = mg - \frac{mv^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

Hence, the magnitude of the force exerted by the seat on the child is less than the weight of the child.

Keep Your Eye on the Ball

Saturday, 30 January, 2021   13:30

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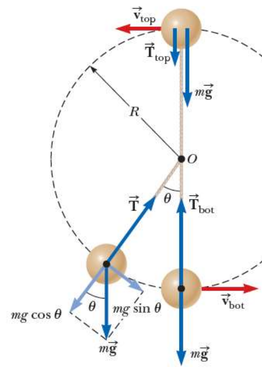
A small sphere of mass (m) is attached to the end of a cord of length (R) and set into motion in a vertical circle about a fixed point O as shown. Determine:

- the tangential acceleration of the sphere,
- the tension in the cord at any instant when the speed of the sphere is (v) and the cord makes an angle θ with the vertical.



Solution

(A) The speed of the sphere is not uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere.

The only forces acting on the sphere are the gravitational force ($m\vec{g}$) exerted by the Earth and the force ($\vec{T}$) exerted by the cord. We resolve ($m\vec{g}$) into a tangential component mg sinθ and a radial component mg cosθ.

Apply Newton's second law to the sphere in the tangential direction:

$$\sum F_t = ma_t \Rightarrow \text{mg}\sin\theta = \text{ma}_t \Rightarrow \text{a}_t = \text{mg}\sin\theta$$

(B) Apply Newton's second law to the forces acting on the sphere in the radial direction, noting that both ($\vec{T}$) and ($\vec{a}_c$) are directed toward O:

$$\sum F_r = ma_r \Rightarrow T - mg\ \cos\theta = \frac{mv^2}{R}$$

So:

$$T = \frac{mv^2}{r} + mg\ \cos\theta$$

Evaluate the tension at the top (θ = 180°, cosθ = -1)

$$T = \frac{mv^2}{r} - mg$$

and at the bottom:(θ = 0°, cosθ = 1)

$$T = \frac{mv^2}{r} + mg$$

$\vec{\mathbf{n}} = \mathbf{0}$

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In the stunt of riding a bicycle in a loop – the – loop, assuming that the loop is a circle with radius $r = 2.7\ m$, what is the least speed v that the bicycle could have at the top of the loop to remain in contact with it there?

(a)

Solution

We can assume that the bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration $\vec{a}$of this particle must have the magnitude $a = \frac{v^2}{r}$and be directed downward, toward the center of the circular loop.

The forces on the particle when it is at the top of the loop are shown. The gravitational force is downward along a y axis; so is the normal force on the particle from the loop (the loop can push down, not pull up); so also is the centripetal acceleration of the particle. Thus, Newton's second law for y components gives us:

$$\sum F_y = n + mg = \frac{mv^2}{r}$$

If the particle has the least speed v needed to remain in contact, then it is on the verge of losing contact with the loop (falling away from the loop), which means that $(n = 0)$at the top of the loop (the particle and loop touch but without any normal force). Substituting 0 for (n), solving for v, and then substituting known values give us:

$$v = \sqrt{gr} = \sqrt{9.8 \times 2.7} = 5.1 m/s$$