

Chapter 7

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ENERGY OF A SYSTEM

Introduction to Energy

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- A variety of problems can be solved with Newton's Laws and associated principles.
- Some problems that could theoretically be solved with Newton's Laws are very difficult in practice.
 - These problems can be made easier with other techniques.
- The concept of energy is one of the most important topics in science and engineering.
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- Energy is not easily defined.

Work, Done by a Constant Force

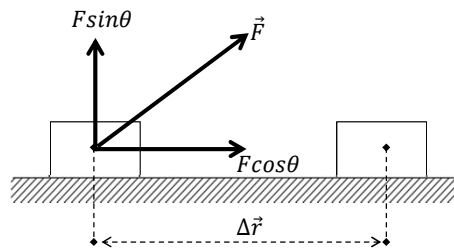
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The work, W , done on a system is the product of the magnitude of the force $|\vec{F}|$, the magnitude of the displacement $|\Delta\vec{r}|$ and $\cos\theta$:

$$W = |\vec{F}| |\Delta\vec{r}| \cos\theta$$

$$W = \vec{F} \cdot \Delta\vec{r}$$

where θ is the angle between the force and the displacement vectors.



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Work, Done by a Constant Force

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- Work is a scalar quantity could be: positive, negative or zero.
- The SI units of work is a Joule (J)

$$1 J = 1 N \cdot m = 1 kg \cdot m^2 / s^2$$

- The meaning of the term work is distinctly different in physics than in everyday meaning.

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Work , Done by a Constant Force

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$$\text{Work is } \begin{cases} +ve ; \text{ when } 0^\circ \leq \theta < 90^\circ ; \text{ energy is transferred to the system} \\ -ve ; \text{ when } 90^\circ < \theta \leq 180^\circ ; \text{ energy is transferred from the system} \\ \text{Zero when } \begin{cases} \theta = 90^\circ \\ \Delta r = 0 \end{cases} ; \text{ energy of the system is constant} \end{cases}$$

Work , Done by a Constant Force

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- Work is positive when projection of \vec{F} onto $\Delta\vec{r}$ is in the same direction as the displacement.
- Work is negative when projection of \vec{F} onto $\Delta\vec{r}$ is in the opposite direction as the displacement.
- A force does no work ($W = 0$) on the object if the force does not move through a displacement.
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.

Work, Done by a Constant Force

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- If the work done on the system is positive, energy is transferred to the system.
- If the work done on the system is negative, energy is transferred from the system.

Work Done by a Varying Force

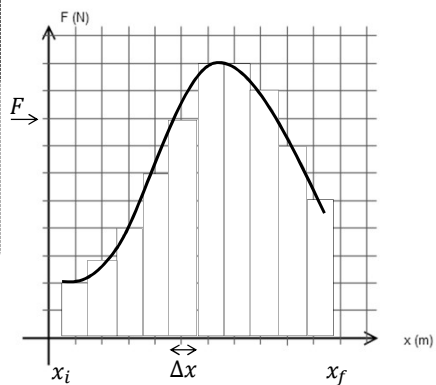
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- To use

$$W = |\vec{F}| |\Delta\vec{r}| \cos\theta$$

- The force must be constant.
- Assume that during a very small displacement, Δx , F is constant.
- For that displacement, $W \cong F\Delta x$
- For all of the intervals,

$$W \cong \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont.

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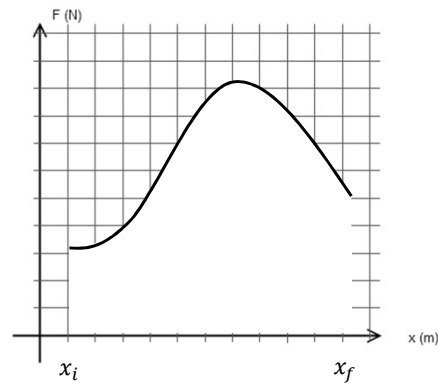
- Let the size of the small displacements approach zero .

$$W = \lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x$$

Therefore,

$$W = \int_{x_i}^{x_f} F_x dx$$

W = the area under the curve
between x_i and x_f .



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Work Done By Multiple Forces

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- If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext.} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

- In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext.} = \int_{r_i}^{r_f} \left(\sum \vec{F} \right) \cdot d\vec{r}$$

- The subscript “ext” indicates the work is done by an *external* agent on the system.

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Work Done by Multiple Forces, cont.

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- If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{ext.} = \sum_{FORC} (\vec{F} \cdot d\vec{r})$$

- Remember work is a scalar, so this is the algebraic sum.

Work - Kinetic Energy Theorem

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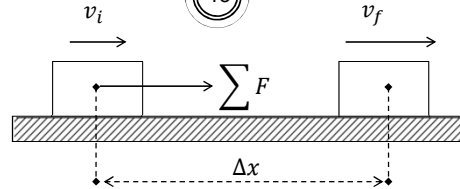
One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess kinetic energy.

A change in kinetic energy is one possible result of doing work to transfer energy into a system.

Work - Kinetic Energy Theorem

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•Calculating the work:

$$W_{ext} = \int_{x_i}^{x_f} \sum F dx = \int_{x_i}^{x_f} m a dx = \int_{x_i}^{x_f} m \frac{dv}{dt} v dt = \int_{v_i}^{v_f} m v dv$$

$$W_{ext} = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$W_{ext} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

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Work - Kinetic Energy Theorem

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Kinetic Energy: the energy of a particle due to its motion.

$$K = \frac{1}{2} m v^2$$

- m is the mass of the particle (kg)
- v is the speed of the particle (m/s)
- K is the kinetic energy
 - A scalar quantity: always positive.
 - SI units (Joule)
 - $1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ J}$

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Work - Kinetic Energy Theorem

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The ($W - \Delta K$) theorem states that:

“When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.”

$$W_{ext} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$W_{ext} = K_f - K_i$$

$$W_{ext} = \Delta K$$

Work-Kinetic Energy Theorem

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- The speed of the system increases if the work done on it is positive.
- The speed of the system decreases if the net work is negative.
 - Also valid for changes in rotational speed
- The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.
- The work-kinetic energy theorem applies to the speed of the system, not its velocity.

Work and Gravitational Potential Energy

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- Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.
- The forces are internal to the system.
- Can be associated with only specific types of forces acting between members of a system
- Potential energy is always associated with a system of two or more interacting objects.

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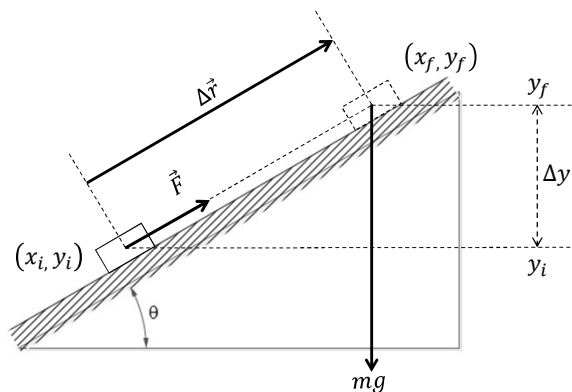
Work and Gravitational Potential Energy

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- Assume the book in the figure is lifted with constant speed.
- There is NO change in kinetic energy.

$$\Delta\vec{r} = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

$$\vec{F}_g = -mg\hat{j}$$



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Work and Gravitational Potential Energy

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The work done by the gravitational force is:

$$W_{mg} = \vec{F}_g \cdot \Delta\vec{r}$$

$$W_{mg} = -mg\hat{j} \cdot [(x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}]$$

$$W_{mg} = -mg(y_f - y_i)$$

$$W_{mg} = -(mgy_f - mgy_i)$$

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Work and Gravitational Potential Energy

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Gravitational Potential Energy

- is the energy stored in an object as the result of its vertical position or height.
- For an object of mass m and at a height y , its gravitational potential energy is given by:

$$U_g = mgy$$

- Is a scalar could be: positive, negative or zero.
- SI units:
 - $kg \cdot \frac{m}{s^2} \cdot m = kg \cdot m^2/s^2 = \text{Joule}$
 - A zero height position must first assigned. Typically, the ground.
 - Any position can be assigned as a zero height position.

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Work and Gravitational Potential Energy

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The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

The work done by the gravitational force along a closed path is zero ($y_f = y_i$).

- A closed path is one for which the beginning point and the endpoint are identical.

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Work and Gravitational Potential Energy

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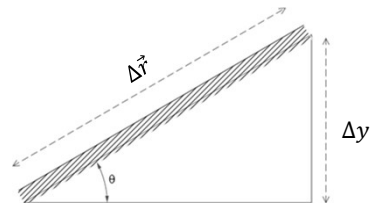
The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

$$W_{mg} = -mg\Delta y$$

$$W_{mg} = -mg\Delta r \sin\theta$$

The work done by the gravitational force is path independent.



$$\sin\theta = \frac{\Delta y}{\Delta r}$$

$$\Delta y = \Delta r \sin\theta$$

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Work and Gravitational Potential Energy

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The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

$$W_{mg} = -(mgy_f - mgy_i)$$

$$W_{mg} = -(U_{gf} - U_{gi})$$

$$W_{mg} = -\Delta U$$

The work done by the gravitational force equals the decrease in potential energy.

Conservative Forces

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The work done by a conservative force on a particle :

- Moving between any two points is independent of the path taken by the particle.
- Moving through any closed path is zero.
- Moving between any two points equals the decrease in potential energy. $W_{c,f} = -\Delta U$

Examples:

- Gravitational force
- Restoring force in a spring
- Electric force
- Magnetic force

Non-conservative Forces

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A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

$$E_{mech.} = K + U$$

K includes the kinetic energy of all moving members of the system.

U includes all types of potential energy in the system.

Conservative Forces and Potential Energy

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Define a potential energy function, U , such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, F , is

$$W_{c.f} = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

ΔU is negative when F and x are in the same direction.

Conservative Forces and Potential Energy

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The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

- The x component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to x .
- Can be extended to three dimensions

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z}$$

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Power

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Power: is the time rate of energy transfer.

Average power: the time rate at which work is being done.

$$P_{avg} = \frac{W}{t} = \frac{E}{t}$$

The instantaneous power is defined as: the power at an instant.

$$P_{inst} = \frac{dW}{dt} = \frac{dE}{dt}$$

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Power

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A scalar quantity could be: positive, negative or zero.

The instantaneous power when Force is constant:

$$P = \vec{F} \cdot \vec{v} = F v \cos\theta$$

This expression for power is valid for any means of energy transfer.

Units of Power

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The SI unit of power is called the watt.

$$1 W = 1 J/s = 1 kg \cdot m^2/s^3$$

A unit of power in the US Customary system is horsepower.

$$1 hp = 746 W$$

Units of power can also be used to express units of work or energy.

$$1 kWh = 1000W \times 3600s = 3.6 \times 10^6 J$$