# Chapter 7

1

## ENERGY OF A SYSTEM



## Introduction to Energy

2

- A variety of problems can be solved with Newton's Laws and associated principles.
- Some problems that could theoretically be solved with Newton's Laws are very difficult in practice.
  - These problems can be made easier with other techniques.
- The concept of energy is one of the most important topics in science and engineering.
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- Energy is not easily defined.

# Work,
# Done by a Constant Force

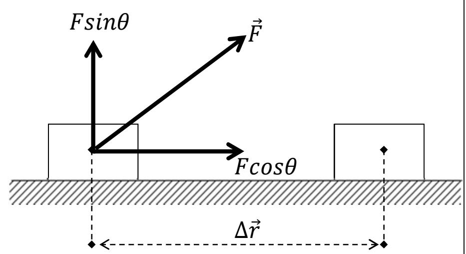
3

The work, W, done on a system is the product of the magnitude of the force $|\vec{F}|$, the magnitude of the displacement $|\Delta\vec{r}|$ and $cos\theta$:

$$W = |\vec{F}|\ |\Delta\vec{r}|\ cos\theta$$

$$W = \vec{F} \cdot \Delta\vec{r}$$

where $\theta$ is the angle between the force and the displacement vectors.

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## Work, Done by a Constant Force

4

- Work is a scalar quantity could be: positive, negative or zero.
- The SI units of work is a Joule (J)

$$1\,J = 1\,N.m = 1\,kg.m^2/s^2$$

- The meaning of the term work is distinctly different in physics than in everyday meaning.

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## Work , Done by a Constant Force

5

$$Work\ is \begin{cases} +ve\ ;\ when\ 0^\circ \le \theta < 90^\circ\ ;\ \text{energy is transferred to the system} \\ -ve\ ;\ when\ 90^\circ < \theta \le 180^\circ\ ;\text{energy is transferred from the system} \\ Zero\ when \begin{cases} \theta = 90^\circ \\ \Delta r = 0 \end{cases} ;\text{energy of the system is constant} \end{cases}$$



## Work , Done by a Constant Force

6

- Work is positive when projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the same direction as the displacement.
- Work is negative when projection of $\vec{F}$ onto $\Delta\vec{r}$ is in the opposite direction as the displacement.
- A force does no work ($W = 0$) on the object if the force does not move through a displacement.
- The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.

# Work, Done by a Constant Force

7

- If the work done on the system is positive, energy is transferred to the system.
- If the work done on the system is negative, energy is transferred from the system.



# Work Done by a Varying Force

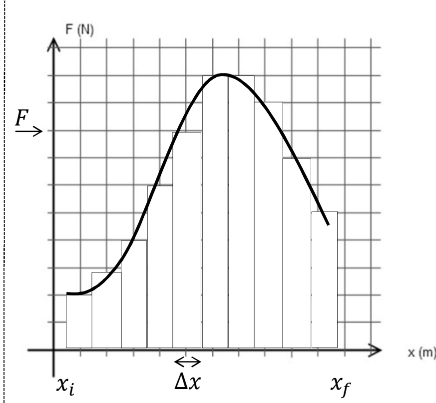
8

- To use

$$W = |\vec{F}|\,|\Delta\vec{r}|\,cos\theta$$

- The force must be constant.
- Assume that during a very small displacement, $\Delta x$ , $F$ is constant.
- For that displacement, $W \cong F\Delta x$
- For all of the intervals,

$$W \cong \sum_{x_i}^{x_f} F_x\,\Delta x$$



# Work Done by a Varying Force, cont.

9

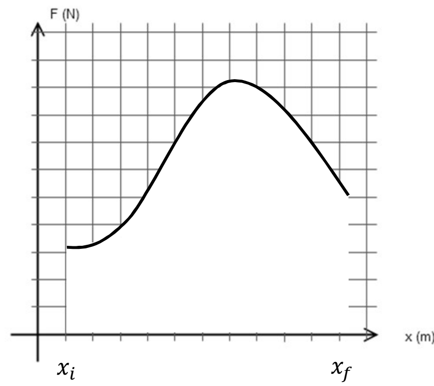
•Let the size of the small displacements approach zero .

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, \Delta x$$

Therefore,

$$W = \int_{x_i}^{x_f} F_x \, dx$$

$W =$ the area under the curve between $x_i$ and $x_f$.






# Work Done By Multiple Forces

10

•If more than one force acts on a system *and the system can be modeled as a particle*, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext.} = \int_{x_i}^{x_f} \left( \sum F_x \right) dx$$

•In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext.} = \int_{r_i}^{r_f} \left( \sum \vec{F} \right) \cdot d\vec{r}$$

•The subscript "ext" indicates the work is done by an *external* agent on the system.

## Work Done by Multiple Forces, cont.

11

- If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{ext.} = \sum_{Forces} (\vec{F} \cdot d\vec{r})$$

  - Remember work is a scalar, so this is the algebraic sum.



## Work - Kinetic Energy Theorem

12

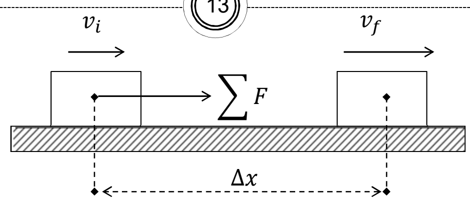
One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess kinetic energy.

A change in kinetic energy is one possible result of doing work to transfer energy into a system.

# Work - Kinetic Energy Theorem

13




- Calculating the work:

$$W_{ext} = \int_{x_i}^{x_f} \sum F\, dx = \int_{x_i}^{x_f} m\, a\, dx = \int_{x_i}^{x_f} m\, \frac{dv}{dt}\, v\, dt = \int_{v_i}^{v_f} m\, v\, dv$$

$$W_{ext} = \frac{1}{2}\, m\left(v_f^2 - v_i^2\right)$$

$$W_{ext} = \frac{1}{2}\, m\, v_f^2 - \frac{1}{2}\, m\, v_i^2$$

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# Work - Kinetic Energy Theorem

14

Kinetic Energy: the energy of a particle due to its motion.

$$K = \frac{1}{2} m v^2$$

- $m$ is the mass of the particle (kg)
- $v$ is the speed of the particle (m/s)
- $K$ is the kinetic energy
  - A scalar quantity: always positive.
  - SI units (Joule)
    - $1\ kg.m^2/s^2 = 1\ J$

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# Work - Kinetic Energy Theorem

15

The $(W - \Delta K)$ theorem states that:

"When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system."

$$W_{ext} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$W_{ext} = K_f - K_i$$

$$W_{ext} = \Delta K$$

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# Work-Kinetic Energy Theorem

16

- The speed of the system increases if the work done on it is positive.
  - The speed of the system decreases if the net work is negative.
    - Also valid for changes in rotational speed
- The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.
- The work-kinetic energy theorem applies to the speed of the system, not its velocity.

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# Work and Gravitational Potential Energy

17

- Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.
- The forces are internal to the system.
- Can be associated with only specific types of forces acting between members of a system
- Potential energy is always associated with a system of two or more interacting objects.



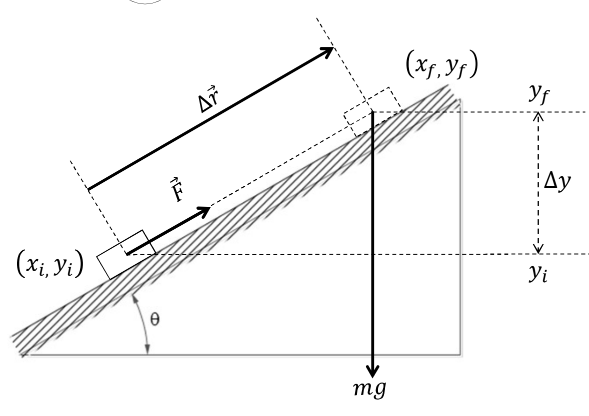
# Work and Gravitational Potential Energy

18

- Assume the book in the figure is lifted with constant speed.
- There is NO change in kinetic energy.

$$\Delta\vec{r} = (x_f - x_i)\hat{\imath} + (y_f - y_i)\hat{\jmath}$$

$$\vec{F}_g = -mg\hat{\jmath}$$



# Work and Gravitational Potential Energy

19

The work done by the gravitational force is:

$$W_{mg} = \vec{F}_g \cdot \Delta\vec{r}$$

$$W_{mg} = -mg\hat{j} \cdot \left[\left(x_f - x_i\right)\hat{\imath} + \left(y_f - y_i\right)\hat{j}\right]$$

$$W_{mg} = -mg\left(y_f - y_i\right)$$

$$W_{mg} = -\left(mgy_f - mgy_i\right)$$



# Work and Gravitational Potential Energy

20

Gravitational Potential Energy

- is the energy stored in an object as the result of its vertical position or height.
- For an object of mass $m$ and at a height $y$, its gravitational potential energy is given by:

$$U_g = mgy$$

- Is a scalar could be: positive, negative or zero.
- SI units:
- $kg.\frac{m}{s^2}.m = kg.m^2/s^2 = Joule$
- A zero height position must first assigned. Typically, the ground.
- Any position can be assigned as a zero height position.

# Work and Gravitational Potential Energy

21

The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

The work done by the gravitational force along a closed path is zero $(y_f = y_i)$.

- A closed path is one for which the beginning point and the endpoint are identical.



# Work and Gravitational Potential Energy

22

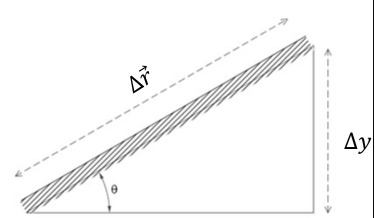
The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

$$W_{mg} = -mg\Delta y$$

$$W_{mg} = -mg\Delta r sin\theta$$

The work done by the gravitational force is path independent.




$$sin\theta = \frac{\Delta y}{\Delta r}$$

$$\Delta y = \Delta r sin\theta$$

## Work and Gravitational Potential Energy

23

The work done by the gravitational force is:

$$W_{mg} = -mg(y_f - y_i)$$

$$W_{mg} = -(mgy_f - mgy_i)$$

$$W_{mg} = -(U_{gf} - U_{gi})$$

$$W_{mg} = -\Delta U$$

The work done by the gravitational force equals the decrease in potential energy.



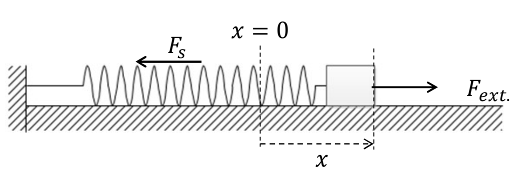
## Work and Elastic Potential Energy

24

Spring force (hooke's law)

The force exerted by the spring is given by hook's law:

$$F_s = -\kappa x$$

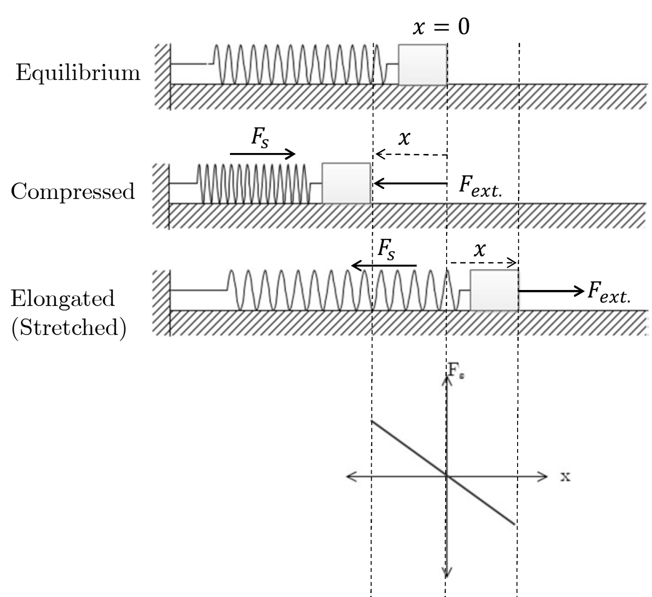


# Work and Elastic Potential Energy

25

- $x$ is the position of the block with respect to the equilibrium position ($x = 0$).
- $\kappa$ is called the spring constant or force constant and measures the stiffness of the spring.
- Negative sign: because the force exerted by the spring is always directed opposite to the displacement from equilibrium.
- The spring force is sometimes called the restoring force.





## Work and Elastic Potential Energy

27

The force varies with position.

Calculate the work as the block moves from $x_i$ to $x_f$.

$$W_s = \int_{x_i}^{x_f} -\kappa x \, dx$$

$$W_s = -\frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

$$W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)$$



## Wok and Elastic Potential Energy

28

The applied force is equal and opposite to the spring force.

For any displacement, the work done by the applied force is

$$W_{ext.} = \frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

# Work and Elastic Potential Energy

29

***Elastic Potential Energy*** is associated with a spring.

The work done by the spring force is:

$$W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)$$

The expression $(\frac{1}{2}\kappa x^2)$ is the elastic potential energy:

$$U_s = \frac{1}{2}\kappa x^2$$

The elastic potential energy can be thought of as the energy stored in the deformed spring.



# Work and Elastic Potential Energy

30

The elastic potential energy stored in a spring:

- Is zero whenever the spring is not deformed (stretched or compressed)(U = 0 when x = 0).
- Is a maximum when the spring has reached its maximum extension or compression.
- Is a scalar quantity.
- Is always positive.
- Has units: $\frac{N}{m}.m^2 = N.m = Joule$

# Work and Elastic Potential Energy

31

The work done by the spring force is:

$$W_s = -\frac{1}{2}\kappa\left(x_f^2 - x_i^2\right)$$

The work done by the spring force along a closed path is zero $(x_f = x_i)$.

The work done by the spring force is path independent.

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# Work and Elastic Potential Energy

32

The work done by the spring force is:

$$W_s = -\left(\frac{1}{2}\kappa x_f^2 - \frac{1}{2}\kappa x_i^2\right)$$

$$W_s = -\left(U_{sf} - U_{si}\right)$$

$$W_s = -\Delta U$$

The work done by the spring force equals the decrease in potential energy.

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# Conservative Forces

33

The work done by a conservative force on a particle :

- Moving between any two points is independent of the path taken by the particle.
- Moving through any closed path is zero.
- Moving between any two points equals the decrease in potential energy. $W_{c.f} = -\Delta U$

Examples:

- Gravitational force
- Restoring force in a spring
- Electric force
- Magnetic force

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# Non-conservative Forces

34

A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

$$E_{mech.} = K + U$$

K includes the kinetic energy of all moving members of the system.

U includes all types of potential energy in the system.

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## Conservative Forces and Potential Energy

35

Define a potential energy function, $U$, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, $F$, is

$$W_{c.f} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

$\Delta U$ is negative when $F$ and $x$ are in the same direction.



## Conservative Forces and Potential Energy

36

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

- The $x$ component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to $x$.
- Can be extended to three dimensions

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

## Power

37

Power: is the time rate of energy transfer.

Average power: the time rate at which work is being done.

$$P_{avg} = \frac{W}{t} = \frac{E}{t}$$

The instantaneous power is defined as: the power at an instant.

$$P_{inst} = \frac{dW}{dt} = \frac{dE}{dt}$$

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## Power

38

A scalar quantity could be: positive, negative or zero.

The instantaneous power when Force is constant:

$$P = \vec{F} \cdot \vec{v} = F\, v\, cos\theta$$

This expression for power is valid for any means of energy transfer.

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# Units of Power

39

The SI unit of power is called the watt.

$$1\,W = 1\,J/s = 1\ kg\,.m^2/s^3$$

A unit of power in the US Customary system is horsepower.

$$1\,hp = 746\,W$$

Units of power can also be used to express units of work or energy.

$$1\,kWh = 1000W \times 3600s = 3.6 \times 10^6 J$$