Chapter 12

1

ELASTICITY

Elasticity

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- We can discuss how objects deform under load conditions.
- An elastic object returns to its original shape when the deforming forces are removed.
- It is possible to change the size and/or shape of the object by applying external forces.
- Internal forces resist the deformation.

Definitions Associated With Deformation

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Stress

- Is proportional to the force causing the deformation
- It is the external force acting on the object per unit cross-sectional area.

Strain

- Is the result of a stress
- Is a measure of the degree of deformation

Elastic Modulus

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•The elastic modulus is the constant of proportionality between the stress and the strain.

- For sufficiently small stresses, the stress is directly proportional to the stress.
- It depends on the material being deformed.
- It also depends on the nature of the deformation.

•The elastic modulus, in general, relates what is done to a solid object to how that object responds.

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

•Various types of deformation have unique elastic moduli.

Three Types of Moduli

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- Young's Modulus
  - Measures the resistance of a solid to a change in its length
- Shear Modulus
  - Measures the resistance of motion of the planes within a solid parallel to each other
- Bulk Modulus
  - Measures the resistance of solids or liquids to changes in their volume

Young's Modulus

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- The bar is stretched by an amount ΔL under the action of the force F.
- The tensile stress is:

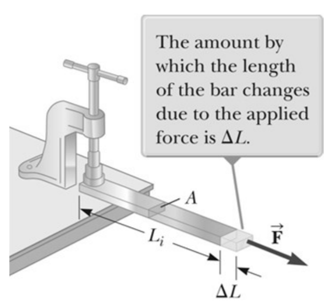
$$\text{tensile stress}=\frac{\text{the external force (F)}}{\text{the cross-sectional area (A)}}$$

  - Units are N/m^2

- The tension strain is:

$$\text{tensile strain}=\frac{\text{the change in length }(\Delta\text{L})}{\text{the original length (L)}}$$

  - Units are: *dimensionless*



Young's Modulus

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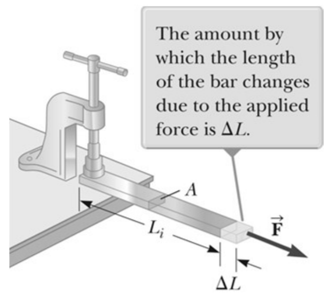
- Young's modulus, Y, is the ratio of those two ratios:

$$\text{Young's modulus} = \frac{\text{tensile stress}}{\text{tensile strain}}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

- Units are N/m^2



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Stress vs. Strain Curve

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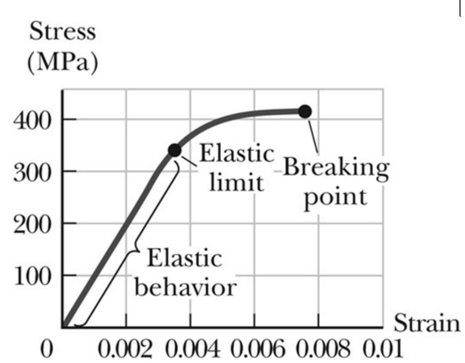
- Experiments show that for certain stresses, the stress is directly proportional to the strain.
- This is the elastic behavior part of the curve.
- The elastic limit is the maximum stress that can be applied to the substance before it becomes permanently deformed.



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Stress vs. Strain Curve, cont

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- When the stress exceeds the elastic limit, the substance will be permanently deformed.
  - The curve is no longer a straight line.
- With additional stress, the material ultimately breaks.

Shear Modulus

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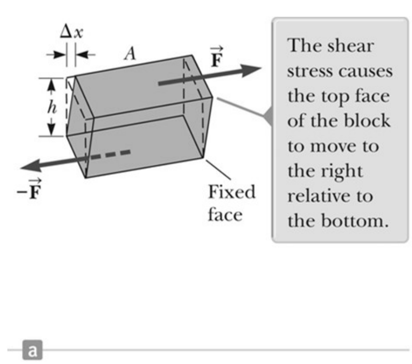
- Another type of deformation occurs when a force acts parallel to one of its faces while the opposite face is held fixed by another force.
- This is called a shear stress.
- For small deformations, no change in volume occurs with this deformation.



Shear Modulus, cont.

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- The shear strain is: $\Delta x/h$.
  - Δx is the horizontal distance the sheared face moves.
  - h is the height of the object.
  - Units are: *dimensionless*
- The shear stress is: F/A.
  - F is the tangential force.
  - A is the area of the face being sheared.
  - Units are N/m^2
- The shear modulus is the ratio of the shear stress to the shear strain.

$$\text{Shear modulus}=\frac{\text{shear stress}}{\text{shear strain}}$$

$$S=\frac{F/A}{\Delta x/h}=\frac{Fh}{A\Delta x}$$

- Units are N / m^2

Bulk Modulus

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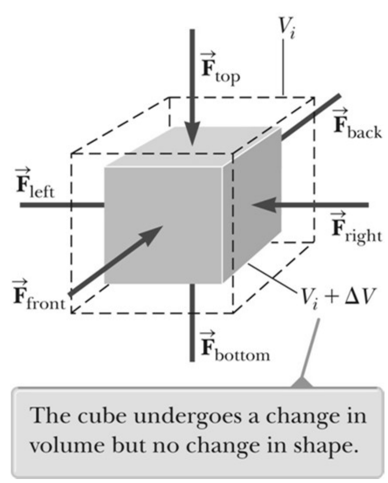
- Another type of deformation occurs when a force of uniform magnitude is applied perpendicularly over the entire surface of the object.
- The object will undergo a change in volume, but not in shape.



Bulk Modulus

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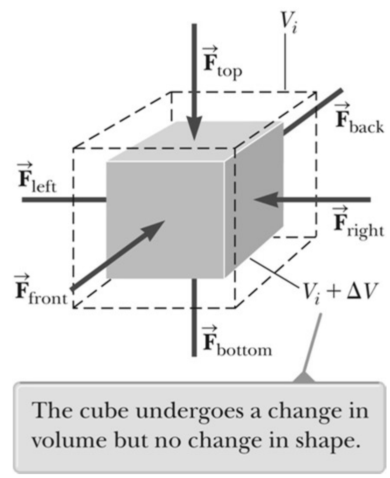
- The volume stress is:

$$\text{volume stress} = \frac{\text{the total force (F)}}{\text{the surface area (A)}}$$

  - This is also called the pressure.
  - Units are N/m^2
- The volume strain is:

$$\text{volume strain} = \frac{\text{the change in volume } (\Delta V)}{\text{the original volume (V)}}$$

  - Units are *dimensionless*



Bulk Modulus, cont.

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- The bulk modulus is the ratio of the volume stress to the volume strain.

$$\text{Bulk modulus} = \frac{\text{volume stress}}{\text{volume strain}}$$

$$B = -\frac{\Delta F/A}{\Delta V/V} = -\frac{V\Delta P}{\Delta V}$$

- Units are N / m^2
- The negative indicates that an increase in pressure will result in a decrease in volume.

Moduli and Types of Materials

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- Both solids and liquids have a bulk modulus.
- Liquids cannot sustain a shearing stress or a tensile stress.
  - If a shearing force or a tensile force is applied to a liquid, the liquid will flow in response.

Moduli Values

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TABLE 12.1 *Typical Values for Elastic Moduli*

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}