Chapter 24

1

Gauss's Law

Electric Flux

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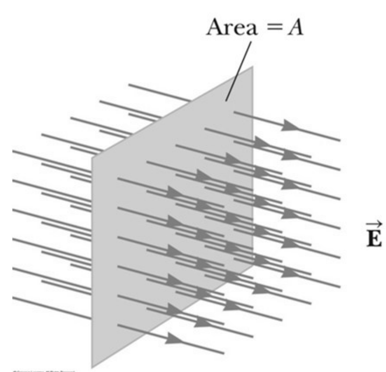
Electric flux is the product of the magnitude of the electric field and the surface area, A, perpendicular to the field.

$$\Phi = EA$$



Electric Flux, General Area

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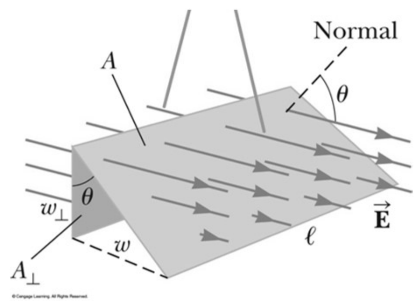
- The electric flux is proportional to the number of electric field lines penetrating some surface.
- The field lines may make some angle θ with the perpendicular to the surface.
- Then:

$$\Phi = EAcos\theta$$

$$\Phi = \vec{E} \cdot \vec{A}$$



Electric Flux, Interpreting the Equation

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Units: $N \cdot m^2/C$

A scalar quantity

θ: the angle between the electric field and the normal to the surface.

- The flux is a maximum when the surface is perpendicular to the field. ($\theta = 0°$)
- The flux is zero when the surface is parallel to the field. ($\theta = 90°$)

Electric Flux, General

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•In the more general case, look at a small area element.

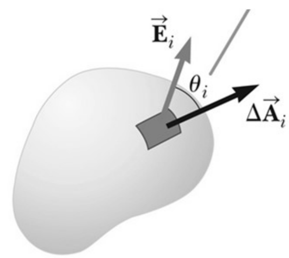
$$\Delta\Phi = E_i \Delta A_i cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

•In general, this becomes

$$\Phi = \lim_{\Delta A_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i$$

○The surface integral means the integral must be evaluated over the surface in question.



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Electric Flux, Closed Surface

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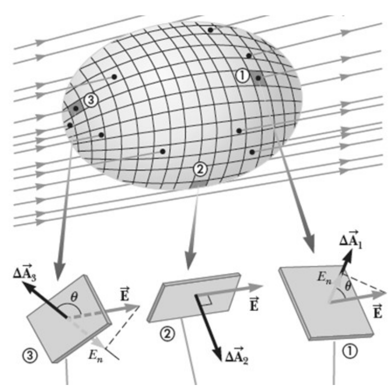
•Assume a closed surface

•The vectors $\Delta\vec{A}_i$ point in different directions.

- At each point, they are perpendicular to the surface.
- By convention, they point outward.



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Flux Through Closed Surface, cont.

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Φ is

(1) $positive$; $0^\circ \le \theta < 90^\circ$;
the field lines are crossing the surface from the inside to the outside

(2) $zero$; $\theta = 90^\circ$;
the field lines graze surface

(3) $negative$; $90^\circ < \theta \le 180^\circ$;
the field lines are crossing the surface from the outside to the inside

Flux Through Closed Surface, final

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- The net flux through the surface is proportional to the net number of lines leaving the surface.
  - This net number of lines is the number of lines leaving the surface minus the number entering the surface.
- If **E** is perpendicular to the surface, then

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i = \oint E\, dA$$

  - The integral is over a closed surface.

Gauss's Law, Introduction

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Gauss's law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface.

The closed surface is often called a gaussian surface.

Gauss's Law – General

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A positive point charge, q, is located at the center of a sphere of radius r.

The magnitude of the electric field everywhere on the surface of the sphere is

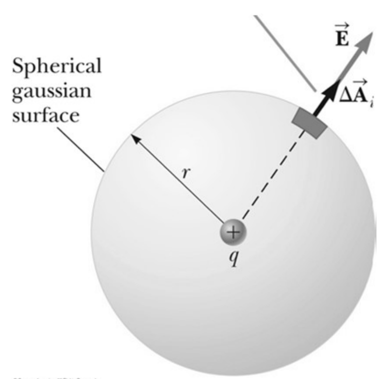
$$E = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r^2}$$

The surface area of a sphere is:

$$A = 4\pi r^2$$

The angle between the directions of $\boldsymbol{E}$ and $\boldsymbol{dA}$ is

$$\theta = 0^\circ$$



Gauss's Law – General, cont.

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•The field lines are directed radially outward and are perpendicular to the surface at every point.

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i = \oint E\, dA = EA$$

•This will be the net flux through the gaussian surface, the sphere of radius r.

$$\Phi = EA = \frac{1}{4\pi\epsilon_\circ} \frac{q}{r^2}\, 4\pi r^2$$

$$\Phi_{net} = \frac{q_{in}}{\epsilon_\circ}$$

Gauss's Law – General

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" The net flux through any closed surface surrounding a point charge, q, is given by

$$\Phi_{net} = \frac{q_{in}}{\epsilon_\circ}$$

and is independent of the shape of that surface."

The net electric flux through a closed surface that surrounds no charge is ZERO.

Gaussian Surface, Example

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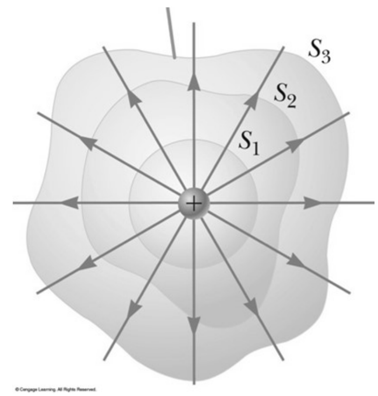
- Closed surfaces of various shapes can surround the charge.
  - Only S_1 is spherical
- Verifies the net flux through any closed surface surrounding a point charge q is given by $q/\epsilon_\circ$ and is independent of the shape of the surface.



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Gaussian Surface, Example 2

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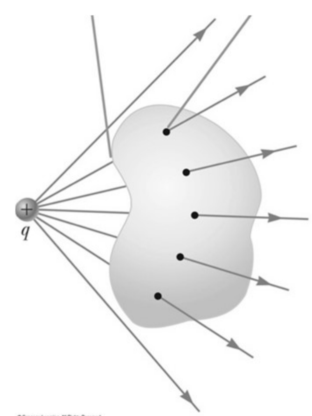
- The charge is outside the closed surface with an arbitrary shape.
- Any field line entering the surface leaves at another point.
- Verifies the electric flux through a closed surface that surrounds no charge is zero.



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Gauss's Law – Final

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The mathematical form of Gauss's law states

$$\Phi_{net} = \oint \vec{E}_i \cdot d\vec{A}_i = \frac{q_{in}}{\epsilon_\circ}$$

- q_{in} is the net charge inside the surface.
- E represents the electric field at any point on the surface.

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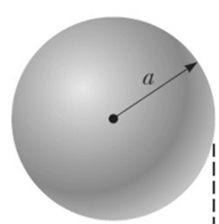
Field Due to a Spherically Symmetric Charge Distribution

- An insulating solid sphere of radius (a) has a uniform volume charge density (ρ) and carries a total positive charge (Q).

a) Calculate the magnitude of the electric field at a point outside the sphere.

b) Calculate the magnitude of the electric field at a point inside the sphere.



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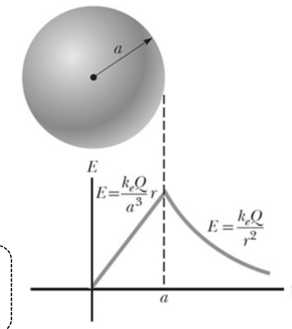
Field Due to a Spherically Symmetric Charge Distribution, FINAL

a) Outside the sphere $(r > a)$

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{r^2}$$

E is Identical to that for a point charge



b) Inside the sphere $(r < a)$

$$E = \frac{\rho}{3\epsilon_\circ} r$$

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{a^3} r$$

E varies linearly with r

$E \rightarrow 0$ as $r \rightarrow 0$

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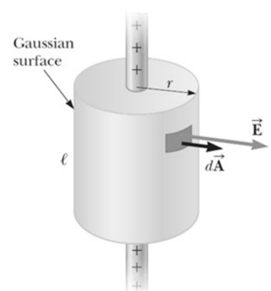
A Cylindrically Symmetric Charge Distribution

•Find the electric field a distance (r) from a line of positive charge of infinite length and constant charge per unit length (λ).



$$E = \frac{1}{2\pi\epsilon_\circ}\frac{\lambda}{r} = \frac{1}{4\pi\epsilon_\circ}\frac{2\lambda}{r}$$

E is inversely proportional to r

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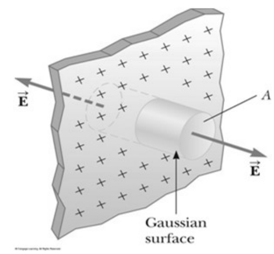
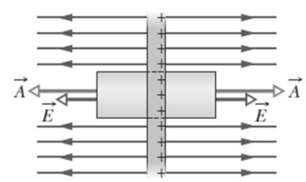
Field Due to a Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density (σ).



$$E = \frac{\sigma}{2\epsilon_\circ}$$



Note, this does not depend on (r). Therefore, the field is uniform everywhere.

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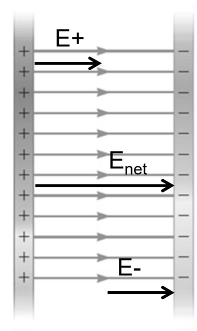
Field Due to two Planes of Charge

Suppose two infinite planes of charge are parallel to each other, one positively charged and the other negatively charged. The surface charge densities of both planes are of the same magnitude. What does the electric field look like in this situation?



$$E_{net} = E_+ + E_- = \frac{\sigma_+}{2\epsilon_\circ} + \frac{\sigma_-}{2\epsilon_\circ}$$

$$E_{net} = \frac{\sigma}{\epsilon_\circ}$$

Uniform E

Properties of a Conductor in Electrostatic Equilibrium

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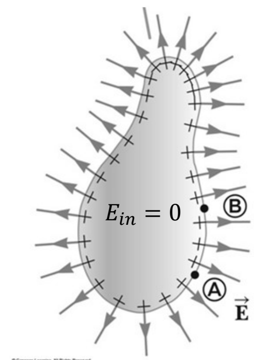
When there is **NO** net motion of charge within a conductor, the conductor is said to be in electrostatic equilibrium, it has the following properties:

1)The electric field is **ZERO** everywhere inside the conductor. Whether the conductor is solid or hollow

2)If the conductor is isolated and carries a charge, **the charge resides on its surface**.



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Properties of a Conductor in Electrostatic Equilibrium

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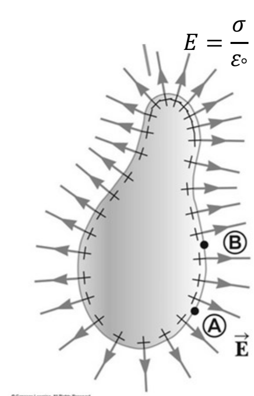
3) The electric field at a point just outside a charged conductor is **perpendicular to the surface** and has a magnitude of: $E = \frac{\sigma}{\varepsilon_\circ}$.

4) On an irregularly shaped conductor, the surface charge density is **greatest** at locations where the radius of curvature is the smallest.



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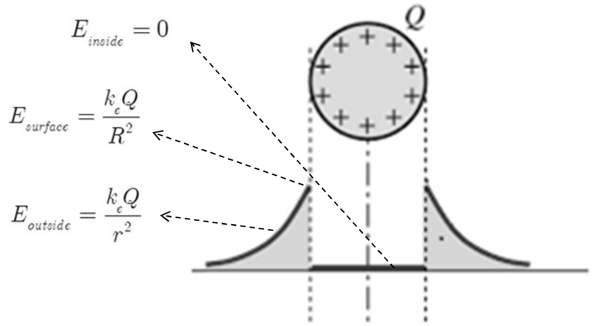
Properties of a Conductor in Electrostatic Equilibrium



R: Radius
Q: Charge
r: distance from the center

Property 1: $\text{Field}_{\text{inside}} = 0$

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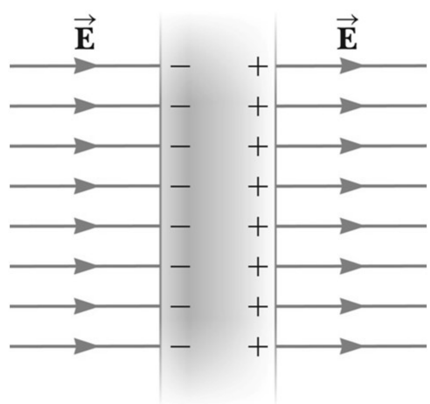
- Consider a conducting slab in an external field.
- If the field inside the conductor were not zero, free electrons in the conductor would experience an electrical force.
- These electrons would accelerate.
- These electrons would not be in equilibrium.
- Therefore, there cannot be a field inside the conductor.



Property 1: $\text{Field}_{\text{inside}} = 0$, cont.

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- Before the external field is applied, free electrons are distributed throughout the conductor.
- When the external field is applied, the electrons redistribute until the magnitude of the internal field equals the magnitude of the external field.
- There is a net field of zero inside the conductor.
- This redistribution takes about 10^{-16} s and can be considered instantaneous.
- If the conductor is hollow, the electric field inside the conductor is also zero.

Property 2: Charge Resides on the Surface

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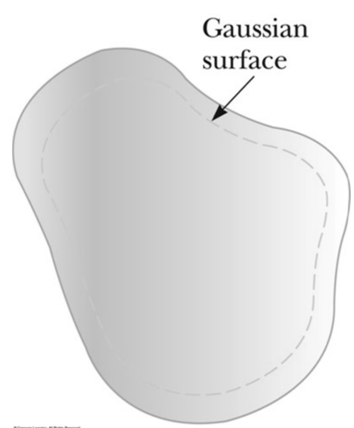
- Choose a Gaussian surface inside but close to the actual surface.
- The electric field inside is zero (property 1).
- There is no net flux through the Gaussian surface.
- Because the Gaussian surface can be as close to the actual surface as desired, there can be no charge inside the surface.



Property 2: Charge Resides on the Surface, cont.

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- Since no net charge can be inside the surface, any net charge must reside ***on*** the surface.
- Gauss's law does not indicate the distribution of these charges, only that it must be on the surface of the conductor.

Property 3: Field's Magnitude and Direction

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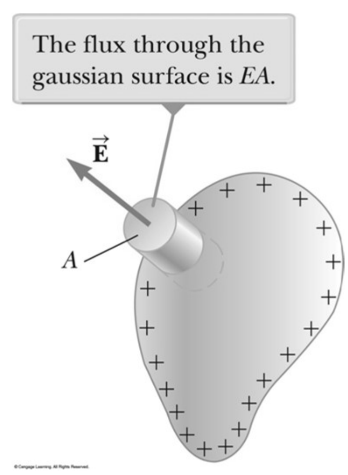
- Choose a cylinder as the Gaussian surface.
- The field must be perpendicular to the surface.
  - If there were a parallel component to ***E*** , charges would experience a force and accelerate along the surface and it would not be in equilibrium.



Property 3: Field's Magnitude and Direction, cont.

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- The net flux through the Gaussian surface is through only the flat face outside the conductor.
  - The field here is perpendicular to the surface.
- Applying Gauss's law

$$\Phi_E = EA = \frac{q_{in}}{\epsilon_\circ}$$

$$E = \frac{q_{in}}{A\epsilon_\circ} = \frac{\sigma}{\epsilon_\circ}$$