Chapter 25

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Electric Potential

Electrical Potential Energy

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When a test charge is placed in an electric field, it experiences a force.

$$\vec{F}_e = q_\circ \vec{E}$$

The electric force is conservative.

The work done by a conservative force:

- Is path independent
- Along a closed path is ZERO
- Equals the decrease in potential energy.

$$W = -\Delta U$$

This will define the electric potential energy .

Electric Potential Energy, cont

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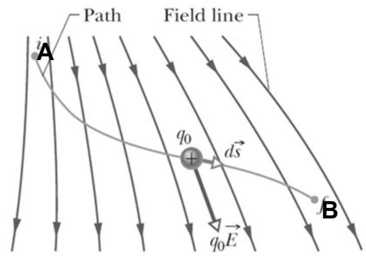
- The work done by the electric force on the charge is:

$$W = \vec{F}_e \cdot d\vec{s} = q_\circ \vec{E} \cdot d\vec{s}$$

- $d\vec{s}$: is an infinitesimal displacement vector that is oriented tangent to the path.



Electric Potential Energy, cont

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- The potential energy of the charge-field system is changed by:

$$\Delta U = -W$$

$$\Delta U = -q_\circ \vec{E} \cdot d\vec{s}$$

- For a finite displacement of the charge from (A) to (B), **the change in potential energy** of the system is

$$\Delta U = U_B - U_A = -q_\circ \int_A^B \vec{E} \cdot d\vec{s}$$

- Because the force is conservative, the line integral does not depend on the path taken by the charge.

Electric Potential

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The electric potential is the potential energy per unit charge:

$$V = \frac{U}{q_\circ}$$

- The potential is a **scalar** quantity.
- SI units:

$$1\, Volt = 1\ J/C$$

Electric Potential

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•As a charged particle moves from (A) to (B) in an electric field, it will experience a **change in electric potential**.

$$\Delta V = V_B - V_A = \frac{\Delta U}{q_\circ}$$

- We often take the value of the potential to be **ZERO** at some convenient point in the field. (Usually at infinity)
- Assume a charge moves in an electric field without any change in its kinetic energy. The work by the field is:

$$W = -\Delta U = -q\ \Delta V$$

Electron-Volts

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- **Another unit of energy** that is commonly used in atomic and nuclear physics is the electron-volt.
- **One electron-volt** is defined as the energy a charge-field system gains or loses when a charge of magnitude (e) (an electron or a proton) is moved through a potential difference of $(1\ volt)$.

$$1\ e.V = 1.6 \times 10^{-19}\ J$$

Calculating the Potential from the Field

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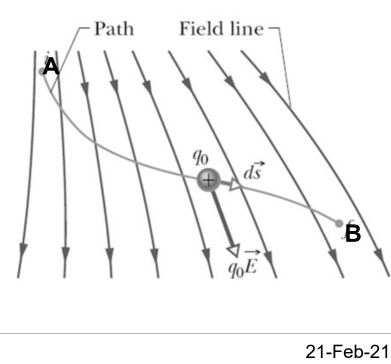
As a charged particle moves from (A) to (B) in an electric field, it will experience a change in electric potential.

$$\Delta V = \frac{\Delta U}{q_\circ}$$

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$



Calculating the Potential from the Field, Uniform E

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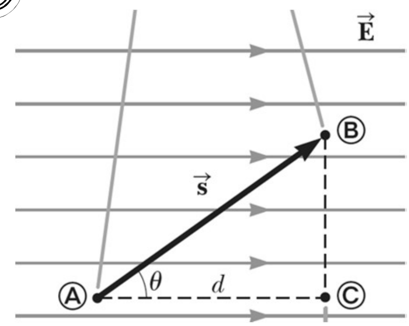
If the electric field is uniform, and the direction of motion between the two points (A) and (B) occurred along a displacement vector ($\vec{s}$) that makes an angle (θ) with the direction of the electric field, then:



$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\int_A^B d\vec{s}$$

$$\Delta V = V_B - V_A = -\vec{E}\cdot\vec{s} = -\left|\vec{E}\right||\vec{s}|\,cos\theta$$

Calculating the Potential from the Field, Uniform E

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- Note that:

$$cos\theta = \frac{d}{s}$$

$$d = s\,cos\theta$$

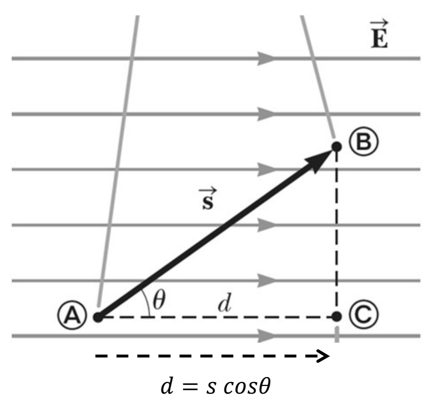
•d: is the displacement from (A) to (C) and is parallel to the field lines.

•Therefore:

$$\Delta V = V_B - V_A = -E\,s\,cos\theta$$

$$\Delta V = -E\,d = V_C - V_A$$

$$\Rightarrow \quad V_B = V_C$$



Equipotential surfaces

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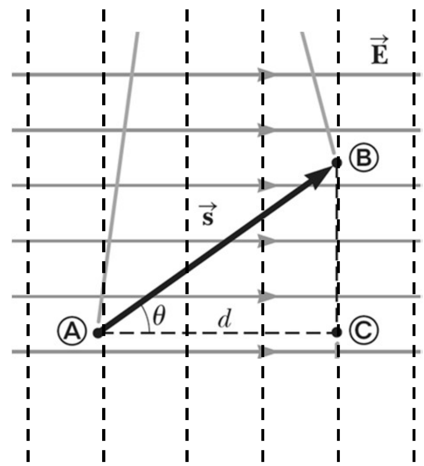
Note that:

$$V_B = V_C$$

Points (B) and (C) are at the same potential.

The name equipotential surface is given to **any surface consisting of a continuous distribution of points having the same electric potential**:

- No net work is done on a charged particle by an electric field when the particle moves between two points on the same equipotential surface.
- Equipotential surfaces are always perpendicular to electric field lines.



Calculating the Potential from the Field, Uniform E

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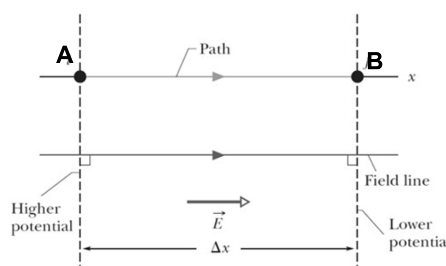
Note that:

The **negative sign** in:

$$V_B - V_A = -\vec{E} \cdot \vec{s} = -|\vec{E}||\vec{s}|\, cos\theta$$

indicates that the electric potential at point (B) is lower than at point (A).

Electric field lines always point in the direction of decreasing electric potential.



Conservation of Mechanical Energy

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Assume a charge moves in an electric field without any change in its kinetic energy. The work by the field is:

$$W = -\Delta U = -q\,\Delta V$$

If a charged particle moves through an electric field and the electric force is the only force acting on it, then the mechanical energy is conserved:

$$\Delta K = -\Delta U = -q\,\Delta V$$

$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = -q\left(V_f - V_i\right)$$

Directions

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If a positive charge is released from rest in an electric field, it accelerates in the direction of the field; then:

- The kinetic energy (ΔK) increases.
- The electric potential energy (ΔU) decreases.
- The electric potential difference (ΔV) decreases.

If a negative charge is released from rest in an electric field, it accelerates in a direction opposite the direction of the field:

- The kinetic energy (ΔK) increases.
- The electric potential energy (ΔU) decreases.
- The electric potential difference (ΔV) increases.

Electric Potential due to a Point Charges

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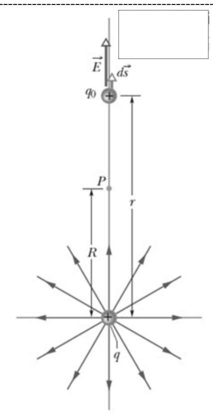
The electric potential at (P) measured with respect to zero potential at infinity will be

$$V_P = \frac{k_e q}{R}$$



Electric Potential due to a Point Charges , cont.

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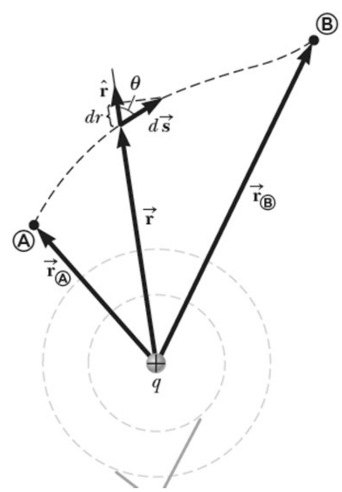
- The potential difference between points A and B will be

$$V_B - V_A = k_e q\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Electric Potential due to a Point Charges, final

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- The electric potential is independent of the path between points *A* and *B*.
- It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$.
- The potential due to a point charge at some point r is

$$V_P = \frac{k_e q}{r}$$

- The potential due to a group of point charges is:

$$V_P = k_e \sum_i \frac{q_i}{r_i}$$

Potential Energy of Multiple Charges

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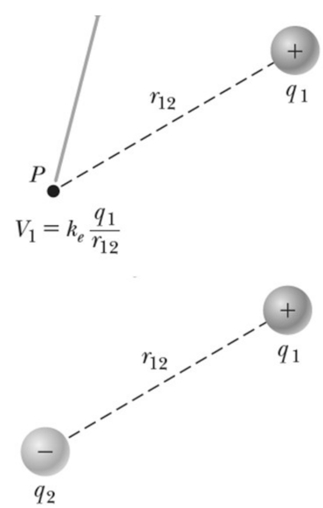
- The potential energy of a system of two charges is:

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

- If the two charges are the same sign, U is positive and work must be done to bring the charges together.
- If the two charges have opposite signs, U is negative and work is done to keep the charges apart.



Potential Energy of Multiple Charges, cont.

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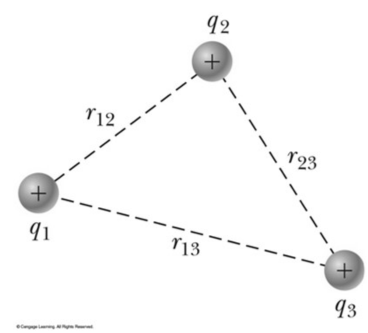
- If there are more than two charges, then find U for each pair of charges and add them.
- For three charges:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$



Obtaining the Value of the Electric Field from the Electric Potential

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Recall that:

$$\Delta V = \int_{V_i}^{V_f} dV = -\int_A^B \vec{E}\cdot d\vec{s} = -\vec{E}\cdot\int_A^B d\vec{s}$$

$$dV = -\vec{E}\cdot d\vec{s}$$

Which tells us how to find ΔV if the electric field $\vec{E}$ is known.
What if the situation is reversed?
Assume, to start, that the field has only an x-component. Then:

$$E_x = -\frac{dV}{dx}$$

Obtaining the Value of E from V, cont.

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Given $V\ (x, y, z)$ you can find E_x, E_y and E_z as partial derivatives:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Equipotential surfaces must always be perpendicular to the electric field lines passing through them. Because:

The motion through a displacement ds along an equipotential surface:

$$dV = 0$$

$$\vec{E} \cdot d\vec{s} = 0$$

$\vec{E}$ and $d\vec{s}$ (and Equipotential surface) must be perpendicular.

E and V for an Infinite Sheet of Charge

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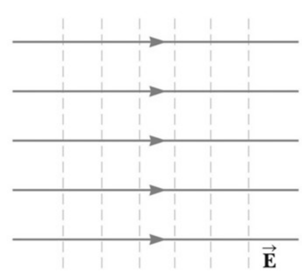
- The equipotential lines are the dashed blue lines.
- The electric field lines are the brown lines.
- The equipotential lines are everywhere perpendicular to the field lines.



E and V for a Point Charge

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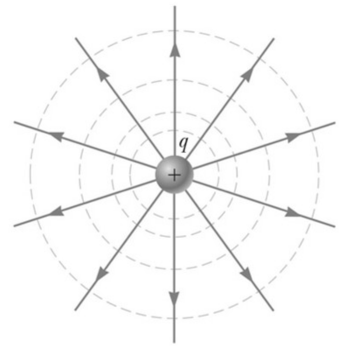
- The equipotential lines are the dashed blue lines.
- The electric field lines are the brown lines.
- The electric field is radial.

$$E_r = -dV/dr$$

- The equipotential lines are everywhere perpendicular to the field lines.



Electric Potential Due to Continuous Charge Distributions

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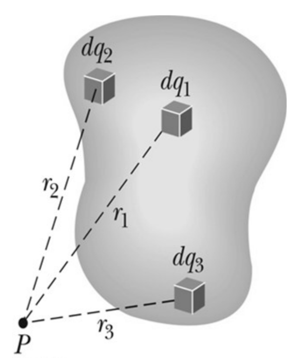
Consider a small charge element dq

Treat it as a point charge. The potential at some point due to this charge element is:

$$dV = k_e \frac{dq}{r}$$

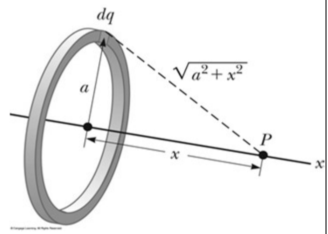
To find the total potential, you need to integrate to include the contributions from all the elements.

$$V = k_e \int \frac{dq}{r}$$



Example 25.5 Electric Potential Due to a Uniformly Charged Ring

A ring of radius (a) carries a uniformly distributed positive total charge (Q). Calculate the electric potential due to the ring at a point (P) lying a distance (x) from its centre along the central axis perpendicular to the plane of the ring.

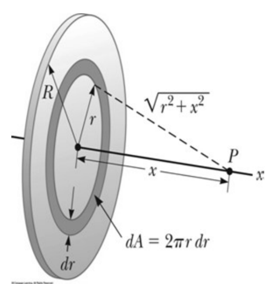


Answer: The potential is given by

$$V = k_e \int_0^Q \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

Example 25.6 Electric Potential Due to a Uniformly Charged Disk

A disk of radius (R) has a uniform surface charge density (σ) . Calculate the electric potential at a point (P) that lies along the central perpendicular axis of the disk and a distance (x) from the center of the disk.



Answer: The potential is given by:

$$V = 2\pi k_e \sigma \int_0^R \frac{r\,dr}{\sqrt{x^2 + r^2}}$$

$$V = 2\pi k_e \sigma \left(\sqrt{R^2 + x^2} - x\right)$$

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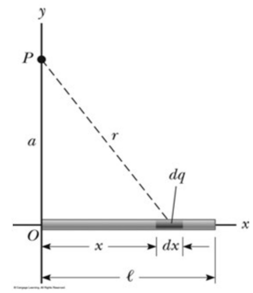
Example 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length (ℓ), located along the x-axis has a total charge (Q) and a uniform linear charge density (λ). Find the electric potential at a point (P) located on the y-axis a distance (a) from the origin.



Answer: The potential is given by:

$$V = k_e\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e\lambda\, ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$

Potential Due to a Charged Conductor

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Consider two points on the surface of the charged conductor as shown.(A and B)

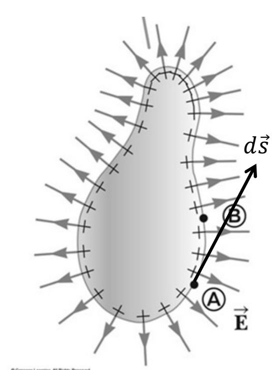
$\vec{E}$ Is always perpendicular to the displacement $d\vec{s}$.

therefore,

$$V_B - V_A = -\vec{E} \cdot d\vec{s} = 0$$

$$V_B = V_A$$

The surface of any charged conductor in electrostatic equilibrium is an **equipotential surface**.



V Due to a Charged Conductor, cont.

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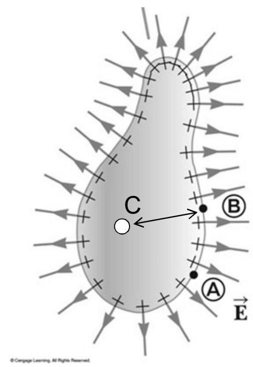
Because the electric field is zero inside the conductor, consider a third point C inside the conductor, the potential difference:

$$V_C - V_B = -\vec{E} \cdot d\vec{s} = 0$$

$$V_A = V_B = V_C$$

We **conclude** that: the electric potential is constant everywhere inside the conductor and equal to the value at the surface.



Irregularly Shaped Objects

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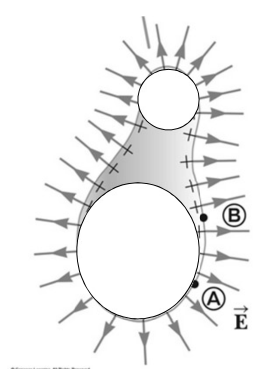
The charge density is high where the radius of curvature is small. And low where the radius of curvature is large:

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi r^2}$$

The electric field is large near the convex points having small radii of curvature and reaches very high values at sharp points.

$$E = \frac{\sigma}{\epsilon_\circ}$$

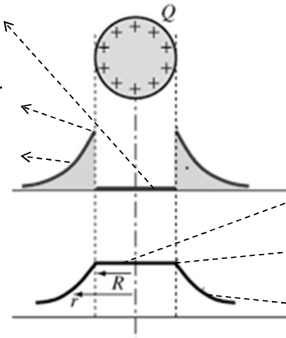


E and V of a Charged Isolated Conducting Sphere (OR) Conducting Spherical Shell

$E_{inside} = 0$

$E_{surface} = \frac{k_e Q}{R^2}$

$E_{outside} = \frac{k_e Q}{r^2}$



$V_{inside} = \frac{k_e Q}{R}$

$V_{surface} = \frac{k_e Q}{R}$

$V_{outside} = \frac{k_e Q}{r}$

R = Radius
Q = Charge
r = distance from the center