Chapter 27

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Current And Resistance

Electric Current

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Electric current is the rate of flow of charge through some region of space.

The SI unit of current is the ampere (A).

$$1\,A = 1\,C/s$$

The symbol for electric current is I.

Average Electric Current

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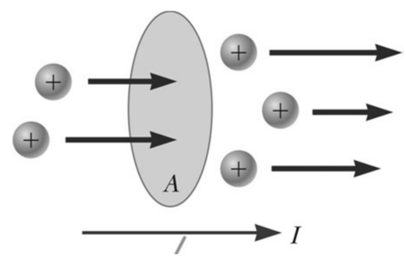
- Assume charges are moving perpendicular to a surface of area A.
- If ΔQ is the amount of charge that passes through A in time Δt, then the **average current** is

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$



Instantaneous Electric Current

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- If the rate at which the charge flows varies with time, the **instantaneous current**, I, is defined as the differential limit of average current as $\Delta t \rightarrow 0$.

$$I_{inst.} = \frac{dQ}{dt}$$

Direction of Current

The charged particles passing through the surface could be positive, negative or both.

It is conventional to assign to the current **the same direction as the flow of positive charges**.

It is common to refer to any moving charge as a *charge carrier.*

Current and Drift Speed

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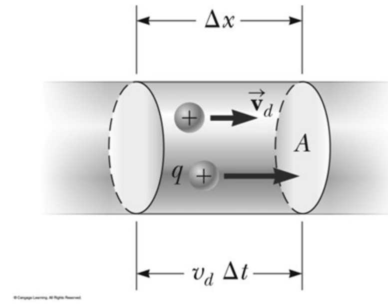
- Charged particles move through a cylindrical conductor of cross-sectional area A.
- (n) is the number of mobile charge carriers per unit volume.

$$Volume = A\Delta x$$

- $(nA\Delta x)$ is the **total number** of charge carriers in a segment.
- The **total charge** is the number of carriers times the charge per carrier, q.

$$\Delta Q = (nA\Delta x)q$$



Current and Drift Speed, cont

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Assume the carriers move with a velocity parallel to the axis of the cylinder such that they experience a displacement in the x-direction.

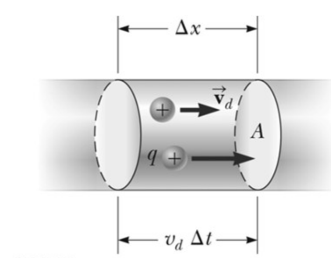
If v_d is the speed at which the carriers move, then

$$v_d = \Delta x/\Delta t \text{ and } \Delta x = v_d\Delta t$$

Rewritten: $\Delta Q = (nAv\_d\Delta t)q$

Finally, current,

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqv_dA$$



v_d is an average speed called the **drift speed.**

Charge Carrier Motion in a Conductor

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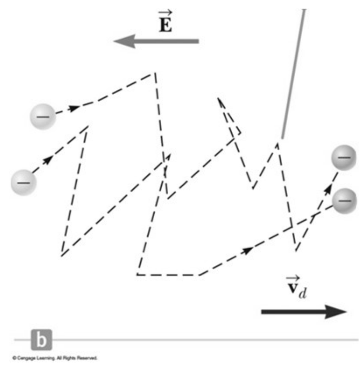
- When a potential difference is applied across the conductor, an electric field is set up in the conductor which exerts an electric force on the electrons.
- The motion of the electrons is no longer random.
- The zigzag black lines represents the motion of a charge carrier in a conductor in the presence of an electric field.
- The sharp changes in direction are due to collisions.
- The net motion of electrons is opposite the direction of the electric field.



Motion of Charge Carriers, cont.

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- In the presence of an electric field, in spite of all the collisions, the charge carriers slowly move along the conductor with a drift velocity,
- The electric field exerts forces on the conduction electrons in the wire.
- These forces cause the electrons to move in the wire and create a current.
- The electrons are already in the wire, they respond to the electric field set up by the battery.
- The battery does not supply the electrons, it only establishes the electric field.

Current Density (J)

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- The **current density** of a conductor is defined as the **current per unit area**.

$$J = \frac{I}{A} = nqv_d$$

- J has SI units of A/m^2
- The current density is in the direction of the positive charge carriers.

Conductivity and resistivity

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- A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor.
- For some materials, the current density is directly proportional to the field.
- The constant of proportionality, σ, is called the **conductivity** of the conductor.
- The inverse of the conductivity is the **resistivity**:

$$\rho = \frac{1}{\sigma}$$

Resistivity has SI units of ohm-meters $(\Omega.m)$

Conductivity has SI units of (ohm-meters)$^{-1}$ $(\Omega.m)^{-1}$

Ohm's Law

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Ohm's law states that for many materials, **the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current**.

Most metals obey ohm's law

Mathematically,

$$J = \sigma E = \frac{E}{\rho}$$

Materials that obey ohm's law are said to be ohmic

Materials that do not obey ohm's law are said to be nonohmic.

Resistance

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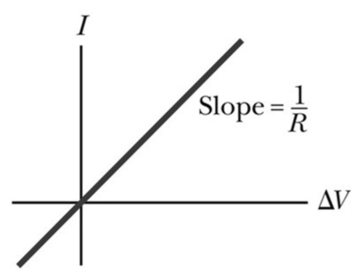
- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.
- The constant of proportionality is called the **resistance** of the conductor.

$$R = \frac{\Delta V}{I}$$

- SI units of resistance are **Ohms** (Ω).

$$1\,\Omega = 1\ V/A$$

- Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor.



Resistance

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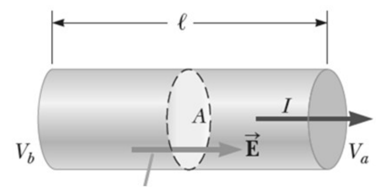
$$J = \sigma E = \frac{E}{\rho}$$

$$\frac{I}{A} = \frac{1}{\rho}\frac{\Delta V}{L}$$

$$\Delta V = \frac{\rho\, L}{A}\, I$$

$$R = \frac{\Delta V}{I} = \frac{\rho\, L}{A}$$



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Resistivity Values

Table 27.2 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and $1.50 \times 10^{-6}\ \Omega \cdot m$.
[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Resistivity, Summary

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- Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.
  - Resistivity is a property of substances.
- The resistance of a material depends on its geometry and its resistivity.
  - Resistance is a property of an object.
- An ideal conductor would have zero resistivity.
- An ideal insulator would have infinite resistivity.

Electrical Conduction – A Model

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Treat a conductor as a regular array of atoms plus a collection of free electrons.

The free electrons are often called conduction electrons.

These electrons become free when the atoms are bound in the solid.

In the absence of an electric field, the motion of the conduction electrons is random.

Their speed is on the order of $10^6\ m/s$.

When an electric field is applied, the conduction electrons are given a drift velocity.

Conduction Model

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- The force experienced by an electron is

$$\vec{F} = q\vec{E}$$

- From Newton's Second Law, the acceleration is

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{q\vec{E}}{m}$$

- Applying a motion equation

Since the initial velocities are random, their average value is zero.

Conduction Model

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- Let τ be the average time interval between successive collisions.
- The average value of the final velocity is the drift velocity.

$$\vec{v}_{f,avg} = \vec{v}_d = \frac{q\vec{E}}{m_e}\tau$$

- This is also related to the current density:

$$J = nqv_d = (nq^2E/m_e)\tau$$

  - n is the number of charge carriers per unit volume.

Conduction Model, final

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- Using Ohm's Law, expressions for the conductivity and resistivity of a conductor can be found:

$$\sigma = \frac{nq^2\tau}{m_e}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

- Note, according to this classical model, the conductivity and the resistivity do not depend on the strength of the field.
  - This feature is characteristic of a conductor obeying Ohm's Law.

Resistance and Temperature

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Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature.

$$\rho = \rho_\circ[1 + \alpha(T - T_\circ)]$$

ρ is the resistivity at some temperature T

$\rho_\circ$ is the resistivity at some reference temperature $T_\circ$

$T_\circ$ is usually taken to be 20 ℃

α is the temperature coefficient of resistivity

SI units of α are ℃$^{-1}$

Temperature Variation of Resistance

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•Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance.

$$R = R_\circ[1 + \alpha(T - T_\circ)]$$

R is the resistivity at some temperature T

$R_\circ$ is the resistivity at some reference temperature $T_\circ$

$T_\circ$ is usually taken to be 20 ℃

α is the temperature coefficient of resistivity

SI units of α are ℃$^{-1}$

Electrical Power

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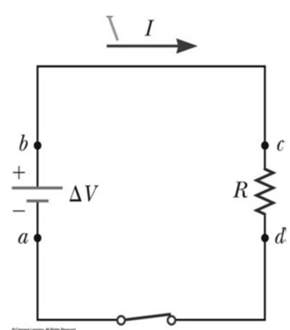
- Assume a circuit as shown.
- The entire circuit is the system.
- As a charge moves from a to b, the electric potential energy of the system increases by $q\Delta V$.
- This electric potential energy is transformed into internal energy in the resistor.
- The **power is** the rate at which the energy is delivered to the resistor.



Electric Power, 2

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- The rate at which the system's potential energy decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.

$$P = \frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = Q\frac{d}{dt}(\Delta V) + \Delta V\frac{d}{dt}(Q)$$

$$P = I\Delta V$$

Electric Power, final

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- Applying Ohm's Law, alternative expressions can be found:

$$P = I\Delta V$$

$$P = I^2 R$$

$$P = \frac{(\Delta V)^2}{R}$$

- Units:
  - I is in A,
  - R is in Ω,
  - ΔV is in V, and
  - P is in W