Chapter 28

1

Direct Current Circuits

Circuit Analysis

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- Simple electric circuits may contain batteries, resistors, and capacitors in various combinations.
- For some circuits, analysis may consist of combining resistors.
- In more complex complicated circuits, Kirchhoff's Rules may be used for analysis.
  - These Rules are based on conservation of energy and conservation of electric charge for isolated systems.

Direct Current

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- When the current in a circuit has a constant direction, the current is called **direct current.**
- Because the potential difference between the terminals of a battery is constant, the battery produces direct current.

Electromotive Force

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- The battery is known as a source of electromotive force (emf) (ε).
- The emf of a battery is the maximum possible voltage the battery can provide between its terminals.
- The phrase electromotive force is an unfortunate historical term, describing not a force, but rather a potential difference in volts.
- The battery will normally be the source of energy in the circuit.
- The positive terminal of the battery is at a higher potential than the negative terminal.
- We consider the wires to have no resistance.

Internal Battery Resistance

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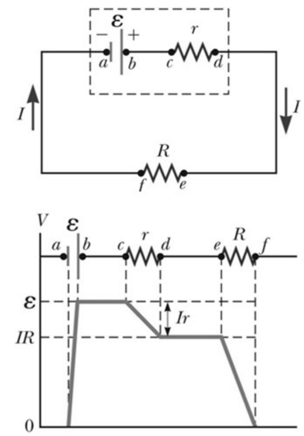
- If the internal resistance is zero, the terminal voltage equals the emf.
- In a real battery, there is internal resistance, r.
- The terminal voltage:

$$\Delta V = V_d - V_a$$
$$\Delta V = \varepsilon - Ir$$

- The emf is equivalent to the *open-circuit* voltage.
  - This is the terminal voltage when no current is in the circuit.
  - This is the voltage labeled on the battery.



Load Resistance

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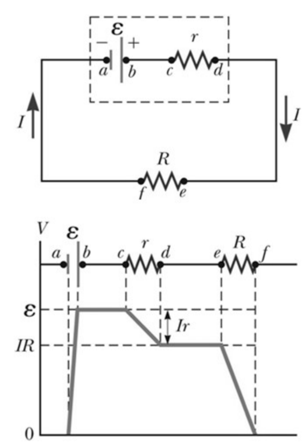
- The actual potential difference between the terminals of the battery depends on the current in the circuit.
- The terminal voltage also equals the voltage across the external resistance.

$$\Delta V = V_e - V_f$$
$$\Delta V = IR$$

  - The load resistance is just the external resistor.
  - In general, the load resistance could be any electrical device.



Current:

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•The current in this simple circuit:

$$V_e - V_f = V_d - V_a$$

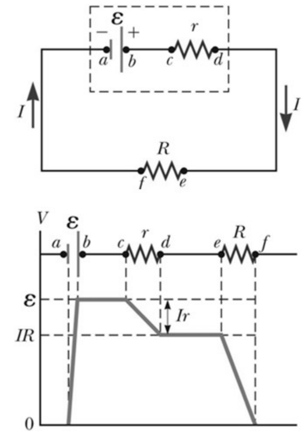
$$IR = \varepsilon - Ir$$

$$I = \frac{\varepsilon}{R+r}$$

•**Note that:**

$\Delta V = \varepsilon$

, $IF \begin{cases} I = 0, \text{ open circuit} \\ r = 0, \text{ NO internal resistance} \end{cases}$



Power

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•The total power output of the battery is

$$P = I\Delta V = I\varepsilon$$

•This power is delivered to the external resistor (I^2R) and to the internal resistor (I^2r).

$$I\varepsilon = I^2R + I^2r$$

Resistors in Series

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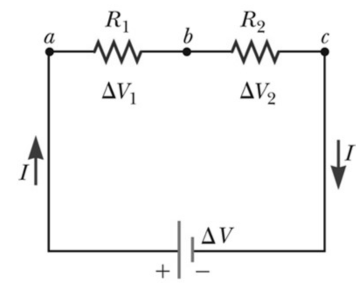
- When two or more resistors are connected end-to-end, they are said to be in series.
- For a series combination of resistors, the currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval.
- The potential difference will divide among the resistors such that the sum of the potential differences across the resistors is equal to the total potential difference across the combination.



Resistors in Series, cont

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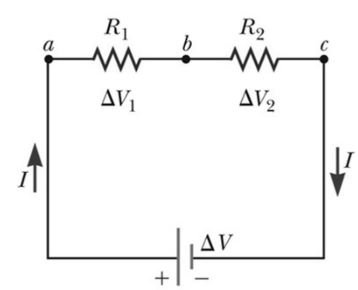
- Currents are the same

$$I = I_1 = I_2 = \cdots$$

- Potentials add

$$\Delta V = \Delta V_1 + \Delta V_2 + \cdots$$

  - Consequence of **Conservation of Energy**



Equivalent Resistance – Series

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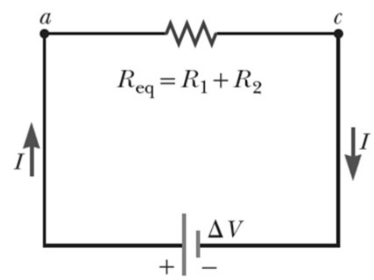
- The equivalent resistance has the same effect on the circuit as the original combination of resistors.

$$R_{eq} = R_1 + R_2 + \cdots$$

- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances.
- If one device in the series circuit creates an open circuit, all devices are inoperative.

The equivalent **is always larger than** the largest resistor in the group.



Resistors in Parallel

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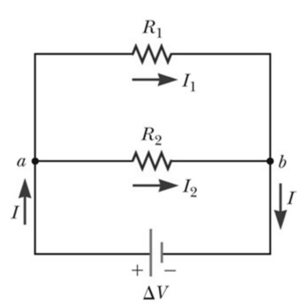
- The potential difference across each resistor is the same because each is connected directly across the battery terminals.

$$\Delta V = \Delta V_1 = \Delta V_2 = \cdots$$

- A **junction** is a point where the current can split.
- The current, I, that enters junction must be equal to the total current leaving that junction.

$$I = I_1 + I_2 + \cdots$$

  - The currents are generally not the same.
  - Consequence of **conservation of electric charge**



Equivalent Resistance – Parallel

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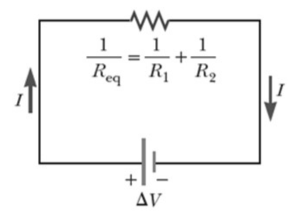
•Equivalent Resistance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

•The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.

- The equivalent **is always less than** the smallest resistor in the group.



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Resistors in Parallel, Final

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•In parallel, each device operates independently of the others so that if one is switched off, the others remain on.

•In parallel, all of the devices operate on the same voltage.

•The current takes all the paths.

- The lower resistance will have higher currents.
- Even very high resistances will have some currents.

•**Household circuits** are wired so that electrical devices are connected in **parallel**.

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Kirchhoff's Rules

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There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.

Two rules, called Kirchhoff's rules, can be used instead.

Kirchhoff's Junction Rule

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- Junction Rule: **the sum of the currents at any junction must equal zero.**

Currents directed into the junction are entered into the equation as $(+I)$ and those leaving as $(-I)$.

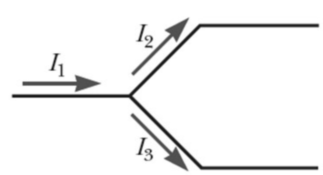
A statement of **Conservation of Charge**

Mathematically,

$$\sum_{Junction} I = 0$$

In the figure:

$$I_1 - I_2 - I_3 = 0$$



Kirchhoff's Loop Rule

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- Loop Rule
  - The sum of the potential differences across all elements around any closed circuit loop must be zero.
    - A statement of **Conservation of Energy**
- Mathematically,

$$\sum_{\substack{closed\\loop}} \Delta V = 0$$

More about the Loop Rule

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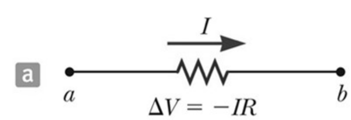
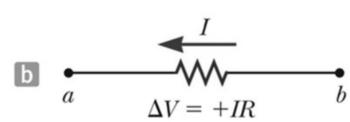
- **Traveling from a to b:**
- In (a), the resistor is traversed in the direction of the current, the potential across the resistor is

$$\Delta V = V_b - V_a = -IR$$

- **Traveling from a to b:**
- In (b), the resistor is traversed in the direction opposite of the current, the potential across the resistor is:

$$\Delta V = V_b - V_a = +IR$$





Loop Rule, final

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•Traveling from a to b:

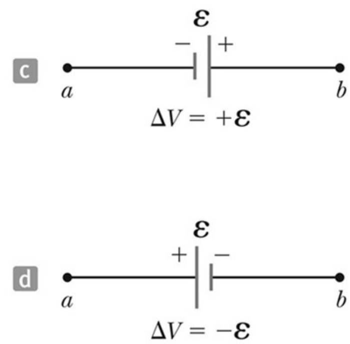
•In (c), the source of emf is traversed in the direction of the emf (from – to +), and the change in the potential difference is

$$\Delta V = V_b - V_a = \varepsilon$$

•Traveling from a to b:

•In (d), the source of emf is traversed in the direction opposite of the emf (from + to -), and the change in the potential difference is

$$\Delta V = V_b - V_a = -\varepsilon$$



RC Circuits

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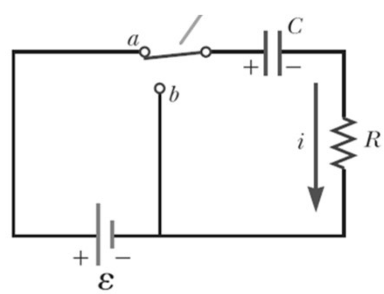
In direct current circuits containing capacitors, the current may vary with time.

The current is still in the same direction.

An RC circuit will contain a series combination of a resistor and a capacitor.



Charging a Capacitor

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- When the circuit is completed, the capacitor starts to charge.
- At the instant the switch is closed, the charge on the capacitor is zero.
- The capacitor continues to charge until it reaches its maximum charge ($Q = C\varepsilon$).
- Once the capacitor is fully charged, the current in the circuit is zero.
- As the plates are being charged, the potential difference across the capacitor increases, such that:

$$\varepsilon = V_C + V_R$$

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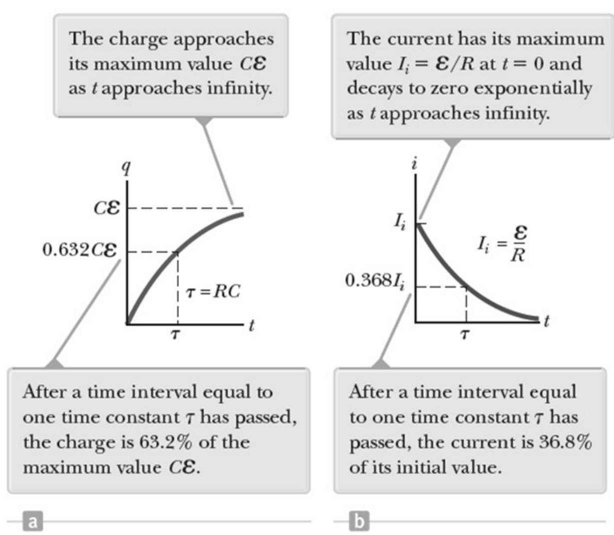
Charging a Capacitor



Time Constant

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τ is the *time constant*

$$\tau = RC$$

The time constant represents the time required for the charge to increase from zero to 63.2% of its maximum.

τ has units of time (s)

Charging a Capacitor, Equations

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When the circuit is completed, the capacitor starts to charge. At the instant the switch is closed ($\boldsymbol{t = 0\,s}$)

The **charge** on the **capacitor** is **zero**:	• $Q_C = 0$
The **potential** difference across the **capacitor** is **zero**:	• $V_C = 0$
The **potential** difference across the **resistor** is **maximum**:	• $V_R = \varepsilon$
The **current** in the circuit is **maximum**:	• $I_{max.} = \frac{\varepsilon}{R}$

Charging a Capacitor , Equations

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After a very long time ($t \to \infty$),

the **charge** on the **capacitor** is **maximum**:	• $Q_{max.} = C\,\varepsilon$
The **potential** difference across the **capacitor** is **maximum**:	• $V_C = \varepsilon$
The **potential** difference across the **resistor** is **zero**:	• $V_R = 0$
The **current** in the circuit is **zero**:	• $I = 0$
The **energy** stored in the capacitor is **maximum**:	• $U_{max.} = \frac{1}{2} Q\,\varepsilon = \frac{1}{2} C\,\varepsilon^2$

Charging a Capacitor , Equations

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While charging the capacitor ($0 < t < \infty$),

The **charge** on the **capacitor increases** with time:	• $Q(t) = Q_{max.}\,(1 - e^{-\frac{t}{\tau}})$
The **potential** difference across the **capacitor increases** with time:	• $V_C(t) = \varepsilon\,(1 - e^{-\frac{t}{\tau}})$
The **energy** stored in the **capacitor increases** with time:	• $U(t) = U_{max.}\,(1 - e^{-\frac{2t}{\tau}})$

Charging a Capacitor , Equations

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The **potential** difference across the **resistor** **decreases** with time:	• $V_R(t) = \varepsilon\, e^{-\frac{t}{\tau}}$
The **current** in the circuit **decreases** with time:	• $I(t) = I_{max.}\, e^{-\frac{t}{\tau}}$

Discharging a Capacitor

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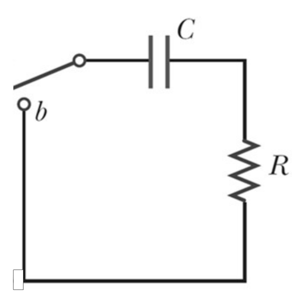
- Imagine that the capacitor in the figure shown is completely charged.
- The initial potential difference across the capacitor is:

$$V_C = \frac{Q_i}{C}$$

- There is zero potential difference across the resistor because $i = 0$.

$$V_R = 0$$

$$I = 0$$



Discharging a Capacitor

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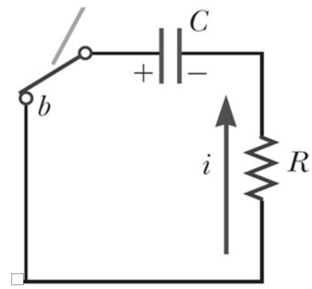
- When the switch is closed, charge begins to flow through resistor R from the right side of the capacitor toward the left side, until the capacitor is fully discharged.
- The voltage across the resistor at any instant **equals** that across the capacitor.
- At some time t during the discharge, the current in the circuit is $I(t)$ and the charge on the capacitor is $Q(t)$.
- As the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.



Discharging a Capacitor

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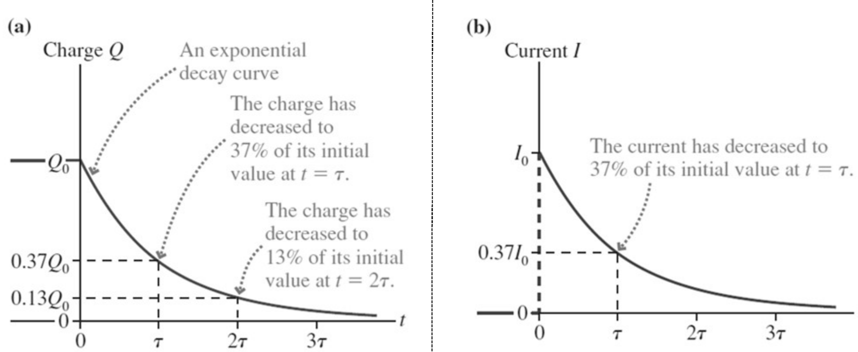

Discharging a Capacitor , Equations

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When the capacitor starts to discharge. At the instant the switch is closed ($\boldsymbol{t = 0\ s}$)	
The **charge** on the **capacitor** is **maximum**:	• $Q_C = Q_{max.}$
The **potential** difference between the plates of the **capacitor** is **maximum**:	• $V_C = \frac{Q_{max.}}{C}$
The **potential** difference between the ends of the **resistor** is **maximum**:	• $V_R = V_C = \frac{Q_{max.}}{C}$
The **current** in the circuit is **maximum**:	• $I_{max.} = \frac{V_R}{R} = \frac{Q_{max.}}{RC}$
The **energy** stored in the capacitor is **maximum**:	• $U_{max.} = \frac{Q^2}{2C}$

Discharging a Capacitor , Equations

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After a very long time ($\boldsymbol{t} \rightarrow \infty$),	
The **charge** on the **capacitor** is **zero** :	• $Q = 0$
The **potential** difference between the plates of the **capacitor** is **zero** :	• $V_C = 0$
The **potential** difference between the ends of the **resistor** is **zero**:	• $V_R = 0$
The **current** in the circuit is **zero**:	• $I = 0$
The **energy** stored in the capacitor is **zero** :	• $U = 0$

Discharging a Capacitor , Equations

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While discharging the capacitor $(\mathbf{0} < \boldsymbol{t} < \infty)$,

The **charge** on the **capacitor decreases** with time:

- $q(t) = Q_{max.}\, e^{-\frac{t}{\tau}}$

The **potential** difference between the plates of the **capacitor decreases** with time:

- $V_C(t) = \frac{Q_{max.}}{C} e^{-\frac{t}{\tau}}$

The **energy** stored in the **capacitor decreases** with time:

- $U(t) = \frac{Q^2}{2C} e^{-\frac{2t}{\tau}}$

Discharging a Capacitor , Equations

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The **potential** difference between the ends of the **resistor decreases** with time:

- $V_R(t) = \frac{Q_{max.}}{C} e^{-\frac{t}{\tau}}$

The **current** in the circuit **decreases** with time:

- $I(t) = \frac{Q_{max.}}{RC} e^{-\frac{t}{\tau}}$