

### **Different Types of Membership Functions**

- Membership function (MF) A function that specifies the degree to which a given input belongs to a set.
- **Degree of membership** The output of a membership function, this value is always limited to between **0** and **1**. Also known as a membership value or membership grade.
- **Membership functions** are used in the **fuzzification** and **defuzzification** steps of a FLS (fuzzy logic system), to map the non-fuzzy input values to fuzzy linguistic terms and vice versa.



- Membership Functions for T (temperature) = too-cold, cold, warm, hot, too-hot.
- There are different forms of membership functions such as:
  - Triangular.
  - Trapezoidal.
  - **Piecewise linear.**
  - Gaussian.
  - Singleton.





- **Fuzzy singleton** A fuzzy set with a membership function that is unity at a one particular point and zero everywhere else.
- Singleton output function- An output function that is given by a spike at a single number rather than a continuous curve. In the Fuzzy Logic Toolbox it is only supported as part of a zero-order Sugeno model.

## **Fuzzy Membership Function: Basic Concepts**

- Support: elements having non-zero degree of membership.
- **Core**: set with elements having degree of 1.
- $\alpha$ -Cut: set of elements with degree >=  $\alpha$ .
- Height: maximum degree of membership.



# Membership Functions in the Fuzzy Logic Toolbox

- A membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse.
- The only condition a membership function must really satisfy is that it must vary between **0** and **1**.
- The Fuzzy Logic Toolbox includes **11** built-in membership function types. These 11 functions are, in turn, built from several basic functions:
  - Piecewise linear functions.
  - Gaussian distribution function.
  - Sigmoid curve.
  - Quadratic polynomial curves.
  - Cubic polynomial curves.
- The simplest membership functions are formed using *straight lines*. These straight line membership functions have the advantage of simplicity.
  - Triangular membership function: trimf.
  - Trapezoidal membership function: trapmf.
- Two membership functions are built on the *Gaussian distribution* curve: a simple Gaussian curve and a two-sided composite of two different Gaussian curves. The two functions are **gaussmf** and **gauss2mf**.
- The *generalized bell membership* function is specified by three parameters and has the function name **gbellmf**.
- Sigmoidal membership function: sigmf.
- Polynomial based curves: Three related membership functions are the Z, S, and Pi curves, all named because of their shape (The functions zmf, smf and pimf).



• Fuzzy Logic Toolbox also allows you to create your *own membership functions.* 

### Example:

```
x = (0:0.1:10)';
y1 = trapmf (x, [2 3 7 9]);
y2 = trapmf (x, [3 4 6 8]);
y3 = trapmf (x, [4 5 5 7]);
y4 = trapmf (x, [5 6 4 6]);
plot (x, [y1 y2 y3 y4]);
Fuzzy logic system
```

- Fuzzy logic system (FLS) can be defined as the **nonlinear mapping** of an input data set to a scalar output data. A FLS consists of four main parts:
  - Fuzzifier (Fuzzification).
  - Rules.
  - Inference engine.
  - Defuzzifier (Defuzzification).
- These components and the general architecture of a FLS are shown in Figure.



#### Fuzzy Logic System

- The process of fuzzy logic:
  - A crisp set of input data are gathered and converted to a fuzzy set using fuzzy linguistic variables, fuzzy linguistic terms and membership functions. This step is known as fuzzification.
  - $\circ~$  An inference is made based on a set of **rules**.
  - The fuzzy output is mapped to a crisp output using the membership functions, in the **defuzzification** step.
- **Fuzzification** The process of generating membership values for a fuzzy variable using membership functions.
- **Defuzzification** The process of transforming a fuzzy output of a fuzzy inference system into a crisp output.

#### **Fuzzy logic algorithm**

- 1) Initialization process:
  - Define the linguistic variables.



- Construct the fuzzy logic membership functions that define the meaning or values of the input and output terms used in the rules.
- Construct the rule base (Break down the control problem into a series of IF X AND Y, THEN Z rules based on the fuzzy logic rules).
- 2) Convert crisp input data to fuzzy values using the membership functions (fuzzification).
- 3) Evaluate the rules in the rule base (inference).
- 4) Combine the results of each rule (inference).
- 5) Convert the output data to non-fuzzy values (defuzzification).
- **Example**: Air conditioner system controlled by a FLS (Figure). The system adjusts the temperature of the room according to the current temperature of the room and the target value. The fuzzy engine periodically compares the room temperature and the target temperature, and produces a command to heat or cool the room.



#### Simple FLS to Control an Air Conditioner

- **Fuzzy Rules:** In fuzzy expert systems, linguistic variables are used in fuzzy rules. For example:
  - IF wind is strong THEN sailing is good.
  - IF project\_duration is long THEN completion\_risk is high.
  - **IF speed is slow THEN stopping\_distance is short.**
- For **Air Conditioner example**, the following rules can be used:
- 1) IF (temperature is cold OR too-cold) AND (target is warm) THEN command is heat.
- 2) IF (temperature is hot OR too-hot) AND (target is warm) THEN command is cool.
- 3) IF (temperature is warm) AND (target is warm) THEN command is nochange.
- **Defuzzification:** The result is a fuzzy value and should be defuzzified to obtain a crisp output. This is the purpose of the defuzzifier component of a FLS. Defuzzification is performed according to the membership function of the output variable.
  - This defuzzification is not part of the 'mathematical fuzzy logic' and various strategies are possible.
  - $\circ~$  The mostly-used algorithms for defuzzification are listed.
    - 1) Finding the center of gravity.
    - 2) Finding the center of gravity for singletons.
    - 3) Finding the average mean.
    - 4) Finding the left most maximum.
    - 5) Finding the right most maximum.