Philadelphia University
Faculty of Engineering

Marking Scheme

Examination Paper

BSc CEE

Signals and Systems *(*650320+*640543)*

Final Exam First semester Date: 30/01/2020

Section 1

Weighting 40% of the module total

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Marking Scheme

Signals and Systems (650320+*640543*)

The presented exam questions are organized to overcome course material through 7 questions.
The *all questions* are compulsory requested to be answered.

Marking Assignments

Question 1 This question is attributed with 10 marks if answered properly; the answers are as following:

1) The fundamental period of the sinusoidal DT signal: $x[n] = \sin\left(\frac{\pi n}{12} + \frac{\pi}{4}\right)$ is

- **a)** 4 **b)** 8
- **c)** 12 **d)** 24

2) Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right) dt$ is

- **a)** 0 **b)** 1
- **c)** -1 **d)** $\pi/2$

3) If $x[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & Otherwise \end{cases}$, describe $x\,[n]$ as superposition of two step functions.

- **a)** $x\,[n] = u\,[n] - u\,[n-5]$ **b)** $x\,[n] = u\,[n-5] - u\,[n]$
- **c)** $x\,[n] = u\,[n-5] + u\,[n]$ **d)** $x\,[n] = u\,[n] + u\,[n-5]$

4) Which one of the following systems is **causal**?

- **a)** $y(t) = x(t) + x(t-3) + x(t^2)$ **b)** $y(n) = x(n+2)$
- **c)** $y(n) = x(2n^2)$ **d)** $y(t) = x(t-1) + x(t-2)$

5) A discrete linear time-invariant system is Bounded Input, Bounded Output (BIBO) **stable** if

a)	$\sum_{k=-\infty}^{\infty} \lvert y[k] \rvert < \infty$
b)	$\sum_{k=-\infty}^{\infty} \lvert h[k] \rvert < \infty$
c)	$\sum_{k=-\infty}^{\infty} \lvert y[k] \rvert \le \sum_{k=-\infty}^{\infty} \lvert h[k] \rvert$
d)	$\sum_{k=-\infty}^{\infty} \lvert y[k] \rvert \le \sum_{k=-\infty}^{\infty} \lvert x[k] \rvert$

6) The step response of a system can be written as

a)	$s(t) = u(t)h(t)$
b)	$s(t) = u(t) * h(t) = \int_{-\infty}^{\infty} \delta(t)h(t-\tau)d\tau$
c)	$s(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau$
d)	**None of above**

7) Determine the convolution sum of two sequences $\mathbf{x(n)} = \{3, 2, 1, 2\}$ and $\mathbf{h(n)} = \{1, 2, 1, 2\}$

- **a)** **y(n) = {3, 8, 8, 12, 9, 4, 4}**
- **b)** **y(n) = {3, 8, 8, 1, 9, 4, 4}**
- **c)** **y(n) = {3, 8, 3, 12, 9, 4, 4}**
- **d)** **y(n) = {3, 8, 8, 12, 9, 1, 4}**

8) The Fourier series of an odd periodic function, contains only

- **a)** **Sine terms** **b)** **Cosine terms**
- **c)** **Odd harmonics** **d)** **Even harmonics**

9) If $\mathbf{x(n)=Ae^{j\omega n}}$ is the input of an **LTI** system and **h(n)** is the response of the system, then what is the output **y(n)** of the system?

- **a)** $H(-\omega)x(n)$ **b)** $-H(\omega)x(n)$
- **c)** $H(\omega)x(n)$ **d)** **None of the mentioned**

10) If $Y(s)$ is the Laplace-transform of the output function, $X(s)$ is the Laplace-transform of the input function and $H(s)$ is the Laplace-transform of system function of the **LTI** system, then $H(s)$ =?

- **a)** $X(s)/Y(s)$ **b)** $Y(s)/X(s)$
- **c)** $Y(s).X(s)$ **d)** **None of the mentioned**

Question 2: This question is attributed with 5 marks if answered properly; the answers are as following:

a) ***(3 marks)***

system	Linear	Time-invariant	Causal
$y(t) = 3x(t)cos(t)$	√	×	√
$y(t) = \int_{t}^{t+1} x(\lambda)d\lambda$	√	√	×

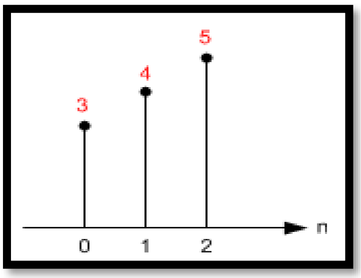
b) ***(2 marks)***

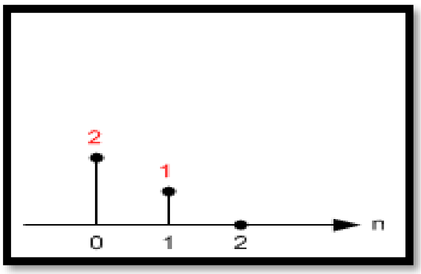
Solution

$$\boxed{h_{overall}(n) = h_1(n) * [(h_2(n) * h_3(n)) + h_4(n)]}$$

Question 3: This question is attributed with 5 marks if answered properly; the answers are as following:

Solution





Input: $x[n]$ Impulse Response: $h[n]$

$$y[n] = \sum x[k] \cdot h[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[0-k] = x[0] \cdot h[0] = 3 \cdot 2 = 6$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[1-k] = x[0] \cdot h[1-0] + x[1] \cdot h[1-1] + \ldots$$

$$= x[0] \cdot h[1] + x[1] \cdot h[0] = 3 \cdot 1 + 4 \cdot 2 = 11$$

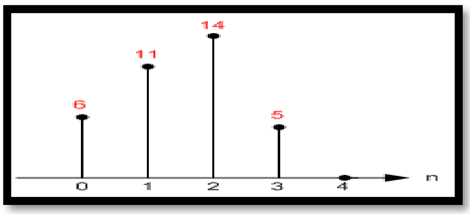
$$y[2] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[2-k] = x[0] \cdot h[2-0] + x[1] \cdot h[2-1] + x[2] \cdot h[2-2] + \ldots$$

$$= x[0] \cdot h[2] + x[1] \cdot h[1] + x[2] \cdot h[0] = 3 \cdot 0 + 4 \cdot 1 + 5 \cdot 2 = 14$$

$$y[3] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[3-k] = x[0] \cdot h[3-0] + x[1] \cdot h[3-1] + x[2] \cdot h[3-2] + x[3] \cdot h[3-3] + \ldots$$

$$= x[0] \cdot h[3] + x[1] \cdot h[2] + x[2] \cdot h[1] + x[3] \cdot h[0] = 0 + 0 + 5 \cdot 1 + 0 = 5$$

$$y[4] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[4-k] = x[0] \cdot h[4-0] + x[1] \cdot h[4-1] + x[2] \cdot h[4-2] + \ldots = 0$$



Output: $y[n]$

Question 4: This question is attributed with 4 marks if answered properly, the answer are as following:

Solution

We have

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{\pi n x}{2} + b_n \sin \frac{\pi n x}{2} \right),$$

where

$$a_0 = \frac{1}{2} \left(\int_{-2}^{0} (-1)\, dx + \int_{0}^{2} 2\, dx \right) = 1,$$

$$a_n = \frac{1}{2} \left(\int_{-2}^{0} (-1) \cos \frac{\pi n x}{2}\, dx + \int_{0}^{2} 2 \cos \frac{\pi n x}{2}\, dx \right) =$$

$$\frac{1}{2} \left((-1) \left[\frac{2}{\pi n} \sin \frac{\pi n x}{2} \right]_{-2}^{0} + 2 \left[\frac{2}{\pi n} \sin \frac{\pi n x}{2} \right]_{0}^{2} \right) = 0, \quad n > 0,$$

and

$$b_n = \frac{1}{2} \left(\int_{-2}^{0} (-1) \sin \frac{\pi n x}{2}\, dx + \int_{0}^{2} 2 \sin \frac{\pi n x}{2}\, dx \right) =$$

$$\frac{1}{2} \left(-(-1) \left[\frac{2}{\pi n} \cos \frac{\pi n x}{2} \right]_{-2}^{0} - 2 \left[\frac{2}{\pi n} \cos \frac{\pi n x}{2} \right]_{0}^{2} \right) =$$

$$\frac{1}{\pi n}(1 - \cos \pi n) - 2 \frac{1}{\pi n}(\cos \pi n - 1) = \frac{3}{\pi n}(1 - (-1)^n).$$

Therefore, we have

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{3}{\pi n}(1 - (-1)^n) \sin \frac{\pi n x}{2}.$$

Question 5: This question is attributed with 9 marks if answered properly, the answers are as following:

a) *(3 marks)*

Solution

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t}\, dt \\ &= \int_{0}^{\infty} e^{-4t} e^{-j\omega t}\, dt + \int_{-\infty}^{0} e^{4t} e^{-j\omega t}\, dt \\ &= \frac{8}{16 + \omega^2} \end{aligned}$$

b) *(3 marks)*

Solution

$$\begin{aligned} x(t) &= \sin(2\pi t) e^{-t} u(t) \\ &= \frac{1}{2j} e^{j2\pi t} e^{-t} u(t) - \frac{1}{2j} e^{-j2\pi t} e^{-t} u(t) \end{aligned}$$

$$e^{-t} u(t) \overset{FT}{\longleftrightarrow} \frac{1}{1 + j\omega}$$

$$e^{j2\pi t} s(t) \overset{FT}{\longleftrightarrow} S(j(\omega - 2\pi))$$

$$X(j\omega) = \frac{1}{2j} \left[\frac{1}{1 + j(\omega - 2\pi)} - \frac{1}{1 + j(\omega + 2\pi)} \right]$$

c) *(3 marks)*

Solution

Taking Fourier Transform on both sides

$$(jw)^2 y(w) + 4jwy(w) + 3y(w) = jwx(w) + 2x(w)$$

$$H(w) = \frac{y(w)}{x(w)} = \frac{2 + jw}{(jw)^2 + 4jw + 3} = \frac{2 + jw}{(1 + jw)(3 + jw)}$$

$$H(w) = \frac{1}{2} \cdot \frac{1}{1 + jw} + \frac{1}{2} \cdot \frac{1}{3 + jw}$$

Taking the inverse Fourier Transform

$$\mathrm{h(t)} = \frac{1}{2} e^{-t} u(t) + \frac{1}{2} e^{-3t} u(t)$$

Question 6: This question is attributed with 3 marks if answered properly, the answers are as following:

Solution

$$x(t) = e^{at}u(t-k)$$

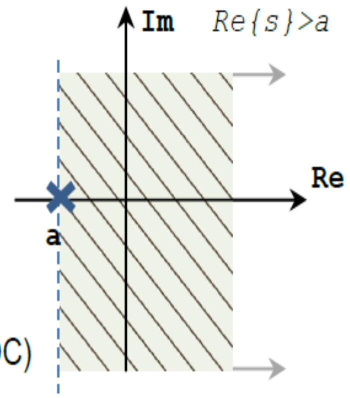
$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt = \int_{k}^{\infty} e^{at}e^{-st}dt = \int_{k}^{\infty} e^{-(s-a)t}dt = -\frac{1}{s-a}e^{-(\sigma+j\omega-a)t}\Big|_{k}^{\infty}$$

$$X(s) = -\frac{1}{s-a}\left(e^{-(\sigma+j\omega-a)\infty} - e^{-(\sigma+j\omega-a)k}\right), \qquad e^{(j\omega)(\pm\infty)} \leq 1$$

$X(s)$ is integrable if $\sigma - a > 0 \rightarrow Re\{s\} > a$

$$\rightarrow X(s) = \frac{e^{ak}}{s-a}e^{-sk}, Re\{s\} > a$$

For the FT to exist, we need $a < 0$ (so that the $j\omega$ axis is included in the ROC)



Question 7: This question is attributed with 4 marks if answered properly, the answers are as following:

Solution

```
% Set the time from -5 to 10 with a sampling rate of 0.002s
t1 = -5:0.002:10;
x1 = 5*exp(-0.2*t1).*sin(2*pi*t1);
subplot(3,1,1);
% plot the first CT signal
plot(t1,x1);
grid on;
xlabel('time (t)');
ylabel('Amplitude');
title('5exp(-0.2t)sin(2\pi t)');
% Set the time from -1 to 15 with a sampling rate of 0.001s
t2 = -1:0.001:15;
x2 = exp((-4*pi-0.5)*t2).*(t2>=0);
% plot the second CT signal
subplot(3,1,2);
plot(t2,x2);
grid on;
xlabel('time (t)')
ylabel('Amplitude');
title('5exp[(-4*pi-0.5)t]u(t)');
% Compute the convolution and plot the result
y = conv(x1, x2);
t3 = 0:length(y)-1;
subplot(3,1,3);
plot(t3,y);
grid on;
xlabel('time (t)')
ylabel('Amplitude');
title('convolution x1 with x2');
```