Instructors:	Philadelphia University	Academic Year: 2023-2024
1. Rola Alseidi	**Faculty of Science**	**Semester:** Fall
2. Ahmad Hamdan	**Department of Mathematics**	**Date:** 2/12/2023
	Midterm Exam	**Course:** Set Theory
		Duration: 60 Min

Name: **I.D. Number:**

Question One: [10 points] **Choose the correct answer**

1. If $A = \{1, 3, 4\}$ and $B = \{x : x \in \mathbb{R}, \text{ and } x^2 - 7x + 12 = 0\}$, then one of the following is **true**

   (A) $A = B$ **(B)** $A \subset B$ **(C)** $\mathcal{P}(B) \subseteq \mathcal{P}(A)$ **(D)** $A \cap B = \phi$ **(E)** $B^c = A$

2. Which of the following are **logically equivalent**

   (i) $A \vee \sim B$
   (ii) $\sim (\sim A \wedge B)$
   (iii) $(A \wedge B) \vee (A \wedge \sim B) \vee (\sim A \wedge \sim B)$
   (iv) $(A \wedge B) \vee (A \wedge \sim B) \vee (\sim A \wedge B)$

   (A) Only i and ii **(B)** Only i, ii and iii **(C)**Only i, ii and iv **(D)** Only ii and iv
   (E) None of them are equivalent

3. Let A and B be any sets. Which of the following rules are **correct**

   (i) $A \cup (\phi \cap A) = B$
   (ii) $A \cap (A \cup B) = B$
   (iii) $(A \cap B) \cup (A \cap B^c) = A$
   (iv) $A - B = A \cap B$

   (A) Only i and ii **(B)** Only i and ii **(C)**Only i, iii **(D)** Only ii and iv **(E)** None of them

4. The **negation** of $\exists x(P(x) \wedge Q(x))$ is**:**

   (A) $\forall x(P(x) \vee Q(x))$ **(B)** $\sim \forall x(P(x) \vee Q(x))$ **(C)** $\exists x \sim P(x) \wedge \exists x Q(x))$
   (D) $\forall x(P(x) \implies \sim Q(x))$ **(E)** $\forall x(\sim P(x) \implies \sim Q(x))$

5. For each real number x, define $B_x = [x^2, x^2 + 1]$. Find $\bigcup_{x \in \mathbb{R}} B_x$:

   (A) ϕ **(B)** $\mathbb{R}$ **(C)** $[0, \infty)$ **(D)** $(-\infty, 0]$ **(E)** None.

Question Two: [10 points (2+4+4)]

1. Without changing its meaning, convert the following sentence in to a sentence having the form " If $P \implies Q$". Then give its converse.

   " For a function f to be continuous, it is sufficient that it is differentiable".

2. Show that the following statement is a tautology.

$$(p \wedge (p \implies q)) \implies q$$

   .

3. Let A, B, and C any subsets of U, prove $(A - B) - C = (A - C) - (B - C)$

Question Three: [8 points (4+4)]

1. Prove that $n^2 + n + 1$ is odd for all $n \in \mathbb{N}$

2. Let $x, y \in \mathbb{R}$ such that $x < 2y$, prove that if $7xy \leq 3x^2 + 2y^2$, then $3x \leq y$

Question Four: [3 points]
Prove that if p is prime number and $p \neq 3$, then $3 \mid (p^2 + 2)$

Good Luck