Department of Civil Engineering

Structural analysis II

Displacement Method of Analysis

Slope- Deflection Equations

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Force method Vs. Displacement method

The force method,

- it requires identifying the unknown redundant forces and then satisfying the structure's compatibility equations.
- This is done by expressing the displacements in terms of the loads by using the load-displacement relations.
- The solution of the resultant equations yields the redundant reactions.
- The equilibrium equations are then used to determine the remaining reactions on the structure.
- its use is limited to structures which are *not* highly indeterminate because much work is required to set up the compatibility equations, and furthermore each equation written involves *all the unknowns,* making it difficult to solve the resulting set of equations.

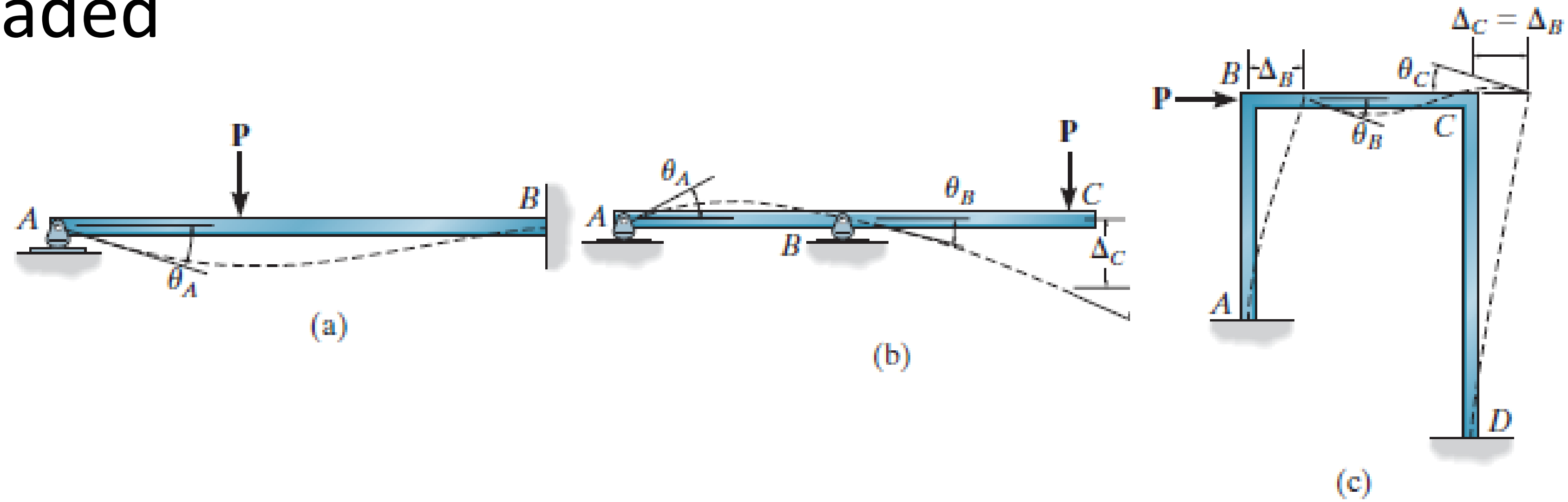
Force method Vs. Displacement method

The displacement method,

- it first requires satisfying equilibrium equations for the structure.
- The unknown displacements are written in terms of the loads by using the load-displacement relations, then these equations are solved for the displacements.
- Once the displacements are obtained, the unknown loads are determined from the compatibility equations using the load-displacement relations.
- it requires less work both to write the necessary equations for the solution of a problem and to solve these equations for the unknown displacements and associated internal loads.
- The method can be easily programmed on a computer and used to analyze a wide range of indeterminate structures.

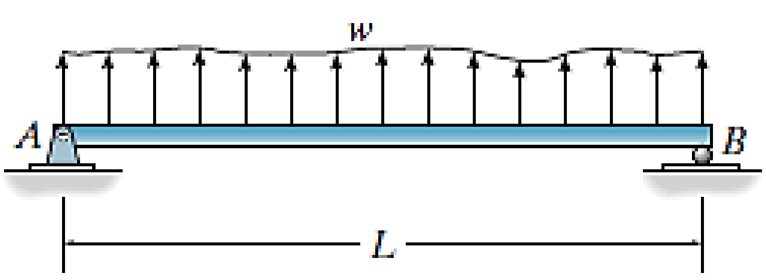
General procedure of Displacement method

Degree of freedom of a system is the unknown displacement of a node when the member is loaded

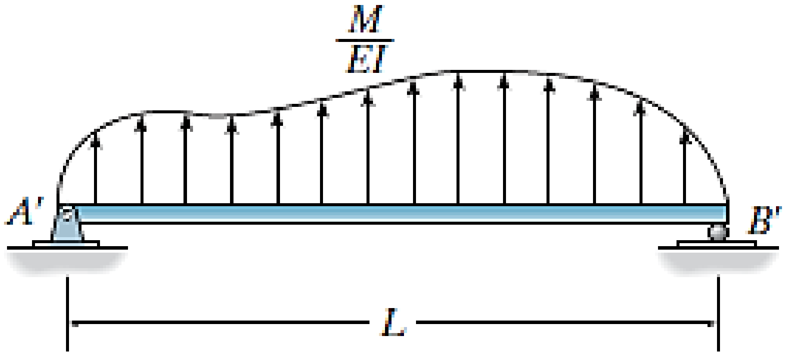


Slope deflection method

- This method relates the unknown slopes and deflections to the applied load on a structure. It is based on using Conjugate beam method.
- The Conjugate beam method requires the same amount of computation as the moment-area theorems to determine a beam's slope or deflection.

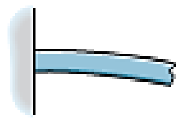
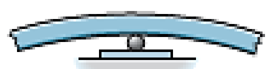
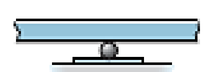


real beam

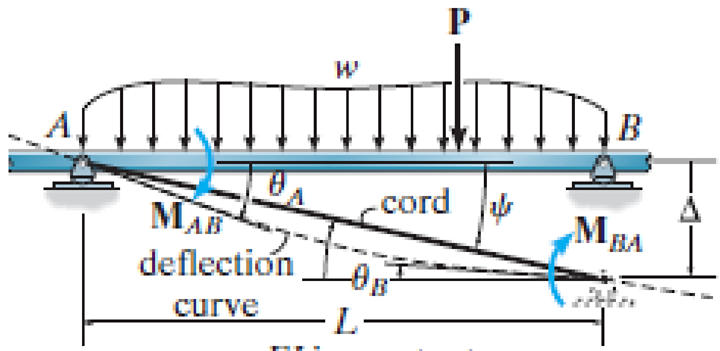


conjugate beam

Remember: Conjugate beam supports

	Real Beam			Conjugate Beam	
1)	θ $\Delta = 0$	 pin		V $M = 0$	 pin
2)	θ $\Delta = 0$	 roller		V $M = 0$	 roller
3)	$\theta = 0$ $\Delta = 0$	 fixed		$V = 0$ $M = 0$	 free
4)	θ Δ	 free		V M	 fixed
5)	θ $\Delta = 0$	 internal pin		V $M = 0$	 hinge
6)	θ $\Delta = 0$	 internal roller		V $M = 0$	 hinge
7)	θ Δ	 hinge		V M	 internal roller

Slope deflection method, General case



EI is constant

Considering Angular displacement at A, θ_A , Load displacement relations are:

$$M_{AB} = \frac{4EI}{L}\theta_A$$

$$M_{BA} = \frac{2EI}{L}\theta_A$$

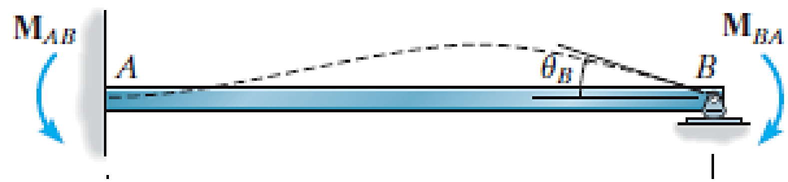
Similarly, considering Angular displacement at B, θ_B, Load displacement relations are:

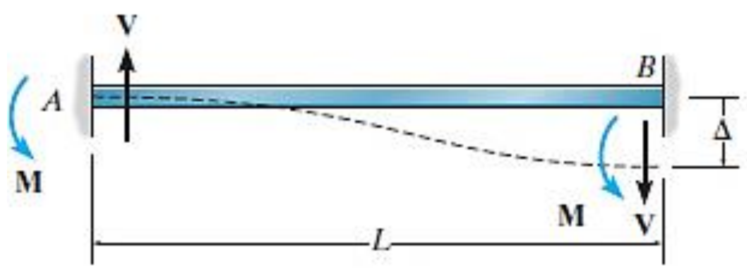
$$M_{BA} = \frac{4EI}{L}\theta_B$$

$$M_{AB} = \frac{2EI}{L}\theta_B$$

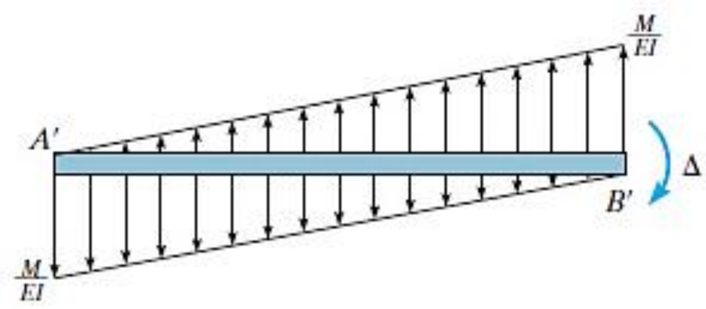
Slope deflection method, General case

Relative Linear Displacement, Δ, If the node *B* of the member is displaced relative to *A*,





real beam



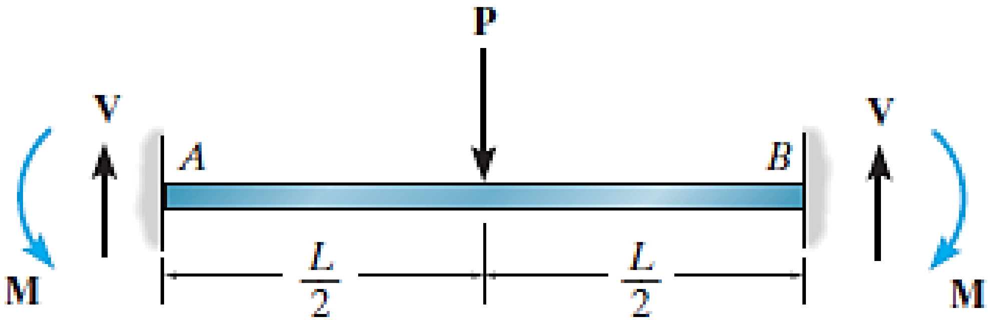
conjugate beam

Load displacement relations are:

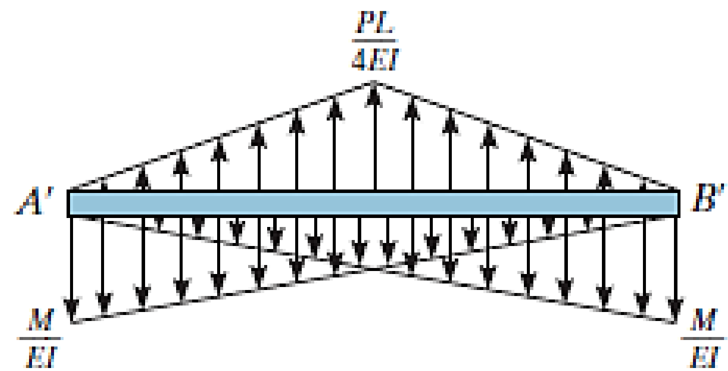
$$M_{AB} = M_{BA} = M = \frac{-6EI}{L^2}\Delta$$

Slope deflection method, General case

Fixed-End Moments,



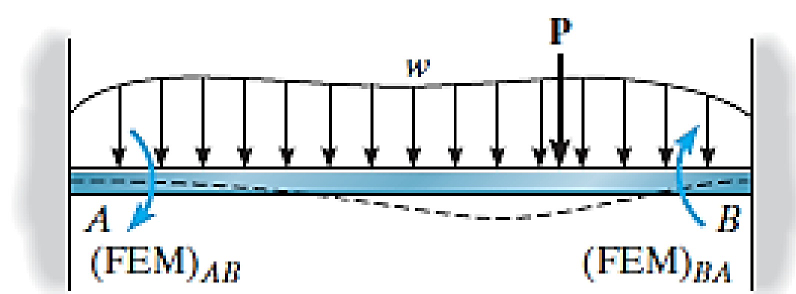
real beam



conjugate beam

For this particular example, $M = \frac{PL}{8}$

$M_{AB} = (\mathrm{FEM})_{AB}$ $\quad$ $M_{BA} = (\mathrm{FEM})_{BA}$



Slope deflection method

Slope deflection equation,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For Internal Span or End Span with Far End Fixed

where

M_N = internal moment in the near end of the span; this moment is *positive clockwise* when acting on the span.

E, k = modulus of elasticity of material and span stiffness $k = I/L$.

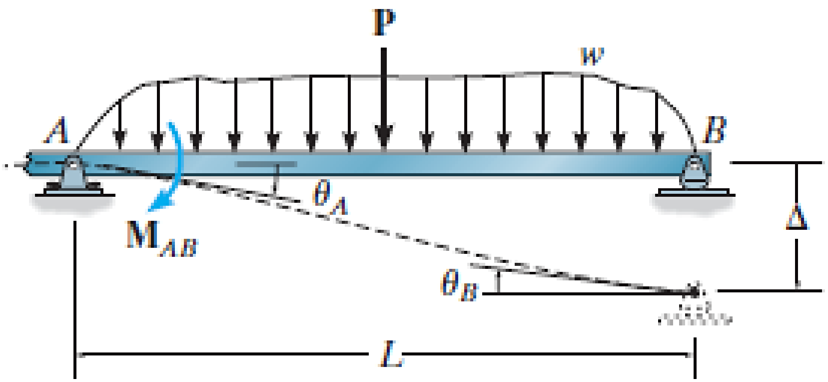
θ_N, θ_F = near- and far-end slopes or angular displacements of the span at the supports; the angles are measured in *radians* and are *positive clockwise*.

ψ = span rotation of its cord due to a linear displacement, that is, $\psi = \Delta/L$; this angle is measured in *radians* and is *positive clockwise*.

$(\text{FEM})_N$ = fixed-end moment at the near-end support; the moment is *positive clockwise* when acting on the span; refer to the table on the inside back cover for various loading conditions.

Slope deflection method

Pin-Supported End Span,



$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

Only for End Span with Far End Pinned or Roller Supported

Procedure of analysis of Beams

1) Degree of freedom

- Label all the supports and joints (nodes) to identify the spans of the beam or frame between the nodes.
- Draw the deflected shape of the structure and identify the number of degrees of freedom.
- Each node can possibly have an angular displacement and a linear displacement.
- Compatibility at the nodes is maintained provided the members that are fixed connected to a node undergo the same displacements as the node.

Procedure of analysis of Beams

2) Slope-Deflection Equations

- If a load exists on the span, compute the related FEMs.
- If a node has a linear displacement Δ, compute $\psi = \Delta/L$ for the adjacent spans.
- Apply the following equation to each end of the span, thereby generating two slope deflection equations for each span.

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For Internal Span or End Span with Far End Fixed

- If a span at the *end* of a continuous beam or frame is pin supported, apply the following equation only to the restrained end, thereby generating *one* slope-deflection equation for the span.

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

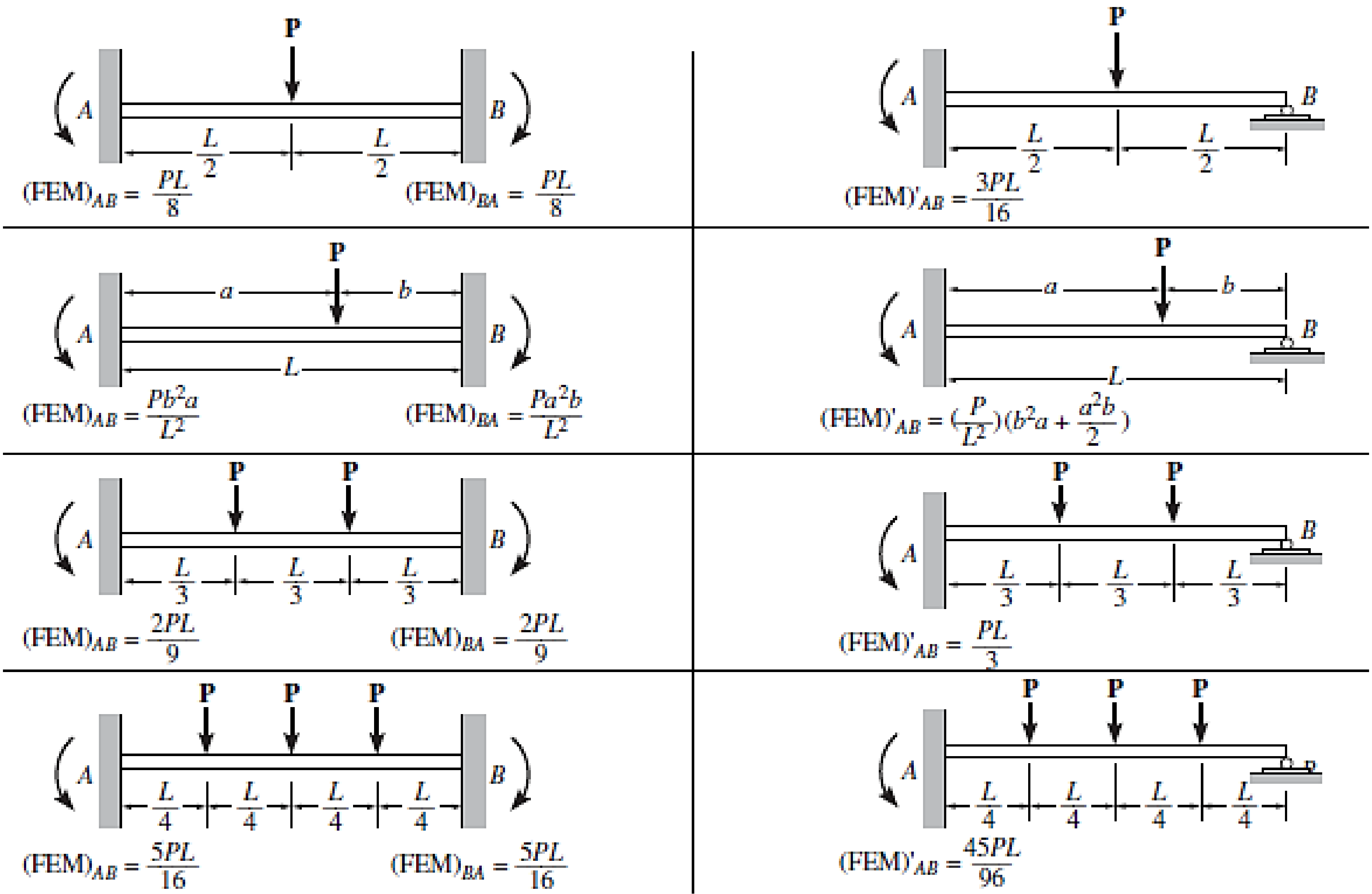
Only for End Span with Far End Pinned or Roller Supported

Procedure of analysis of Beams

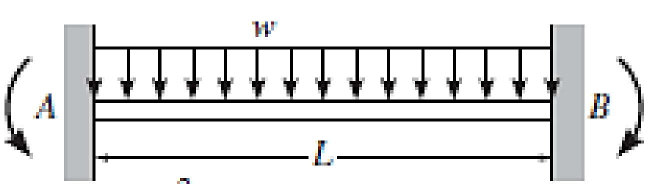
3) Equilibrium Equations

- Write an equilibrium equation for each unknown degree of freedom for the structure in terms of unknown internal moments as specified by the slope-deflection equations.
- For beams and frames, write the moment equation of equilibrium at each support, and for frames also write joint moment equations of equilibrium.
- If the frame sidesways or deflects horizontally, column shears should be related to the moments at the ends of the column.
- Substitute the slope-deflection equations into the equilibrium equations and solve for the unknown joint displacements.
- Substitute the displacements into the slope-deflection equations to determine the internal moments at the ends of each member.

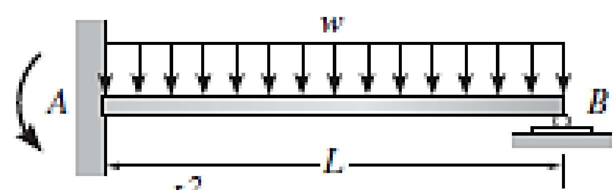
Fixed end moments due to point loads



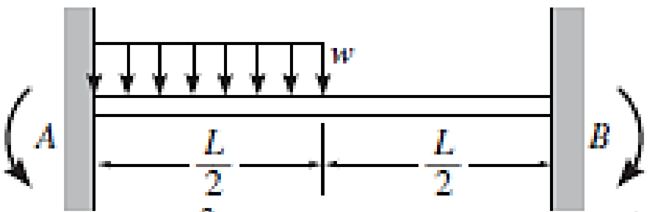
Fixed end moments due to distributed loads



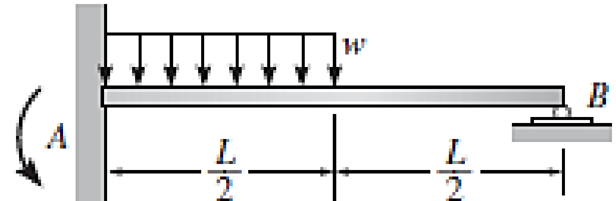
$(FEM)_{AB} = \frac{wL^2}{12}$ $\quad (FEM)_{BA} = \frac{wL^2}{12}$



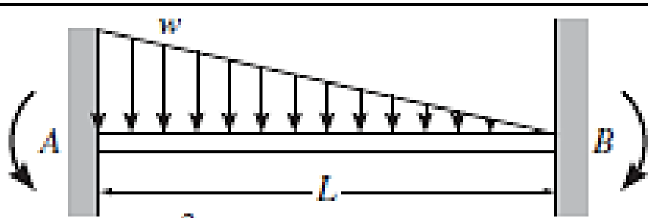
$(FEM)'_{AB} = \frac{wL^2}{8}$



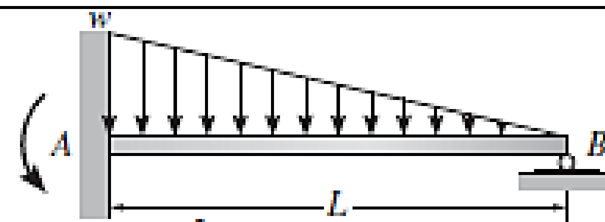
$(FEM)_{AB} = \frac{11wL^2}{192}$ $\quad (FEM)_{BA} = \frac{5wL^2}{192}$



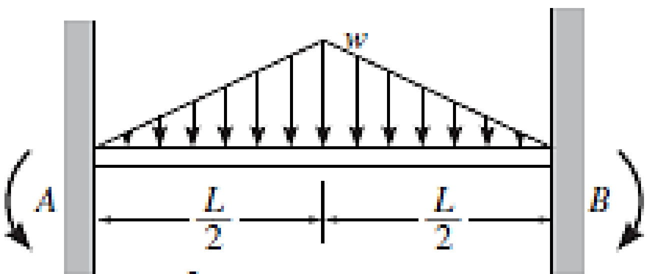
$(FEM)'_{AB} = \frac{9wL^2}{128}$



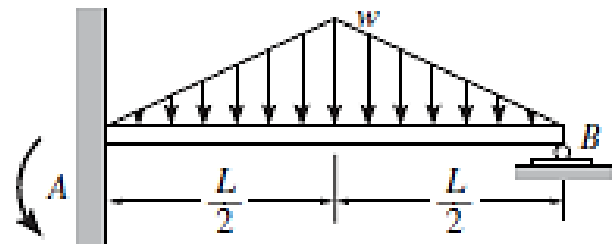
$(FEM)_{AB} = \frac{wL^2}{20}$ $\quad (FEM)_{BA} = \frac{wL^2}{30}$



$(FEM)'_{AB} = \frac{wL^2}{15}$



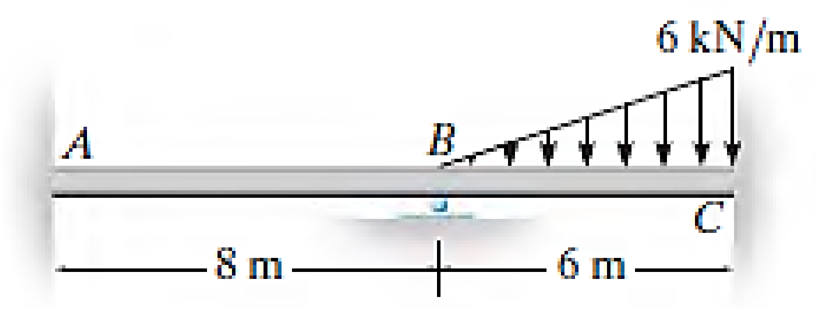
$(FEM)_{AB} = \frac{5wL^2}{96}$ $\quad (FEM)_{BA} = \frac{5wL^2}{96}$

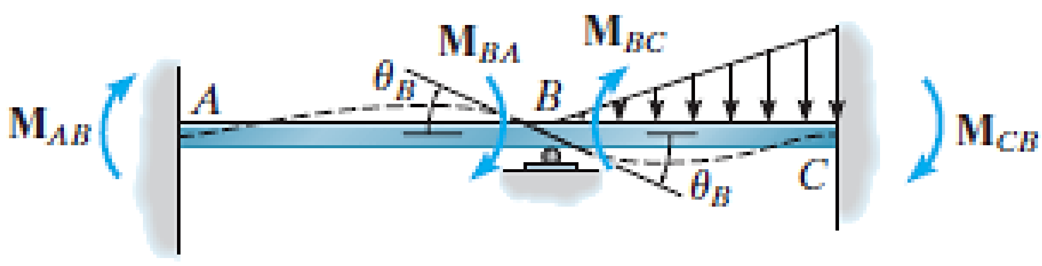


$(FEM)'_{AB} = \frac{5wL^2}{64}$

Example:

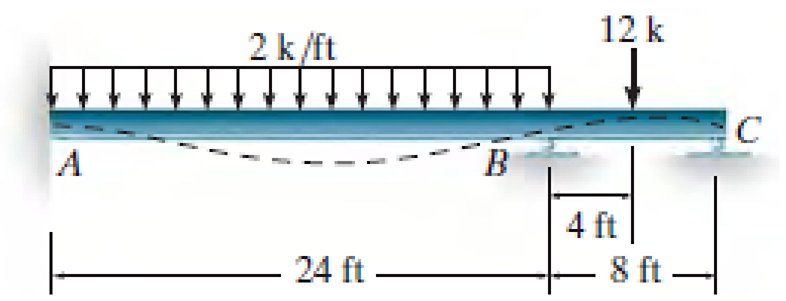
Draw the shear and moment diagrams for the beam. *EI* is constant.





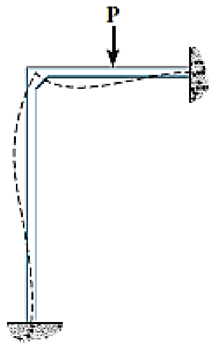
Example:

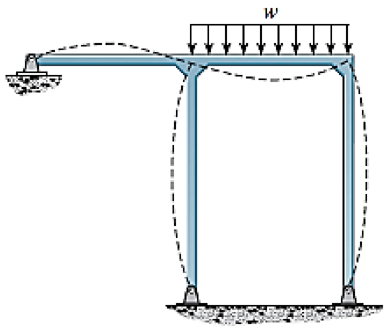
Draw the shear and moment diagrams for the beam. *EI* is constant.



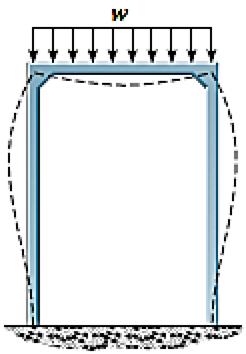
Analysis of frames: No sidesway

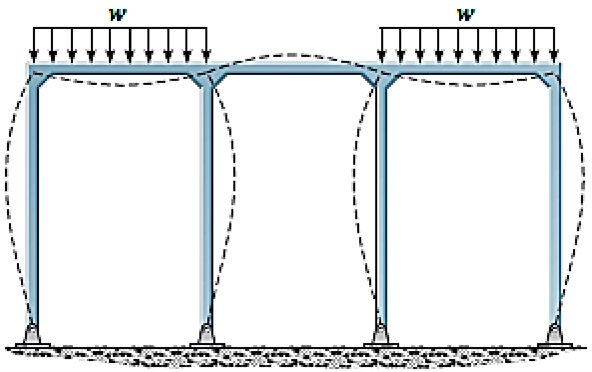
- A frame will not sidesway, or be displaced to the left or right, provided it is properly restrained.





- No sidesway will occur in an unrestrained frame provided it is symmetric with respect to both loading and geometry.

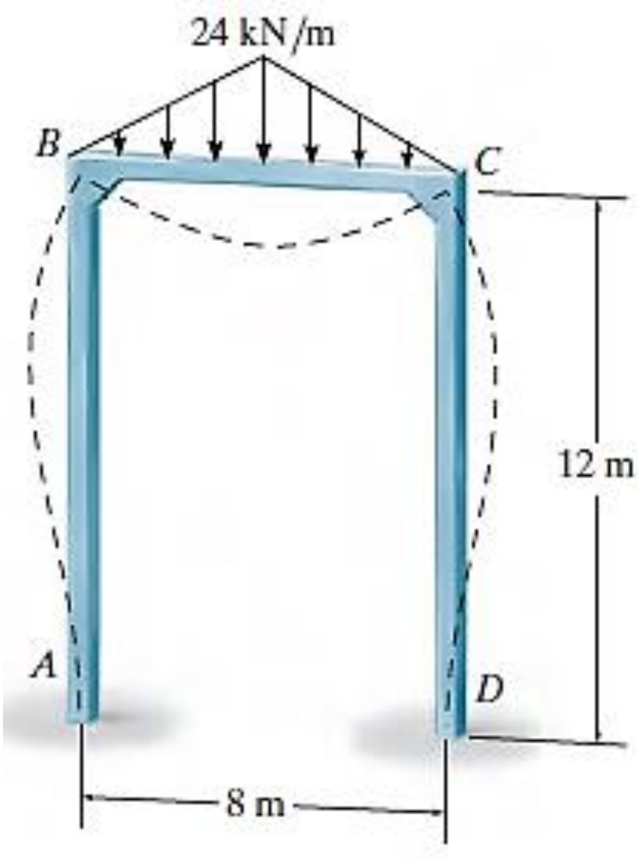




- For both cases bending does not cause the joints to have a linear displacement, therefore the term ψ in the slope-deflection equations is equal to zero.

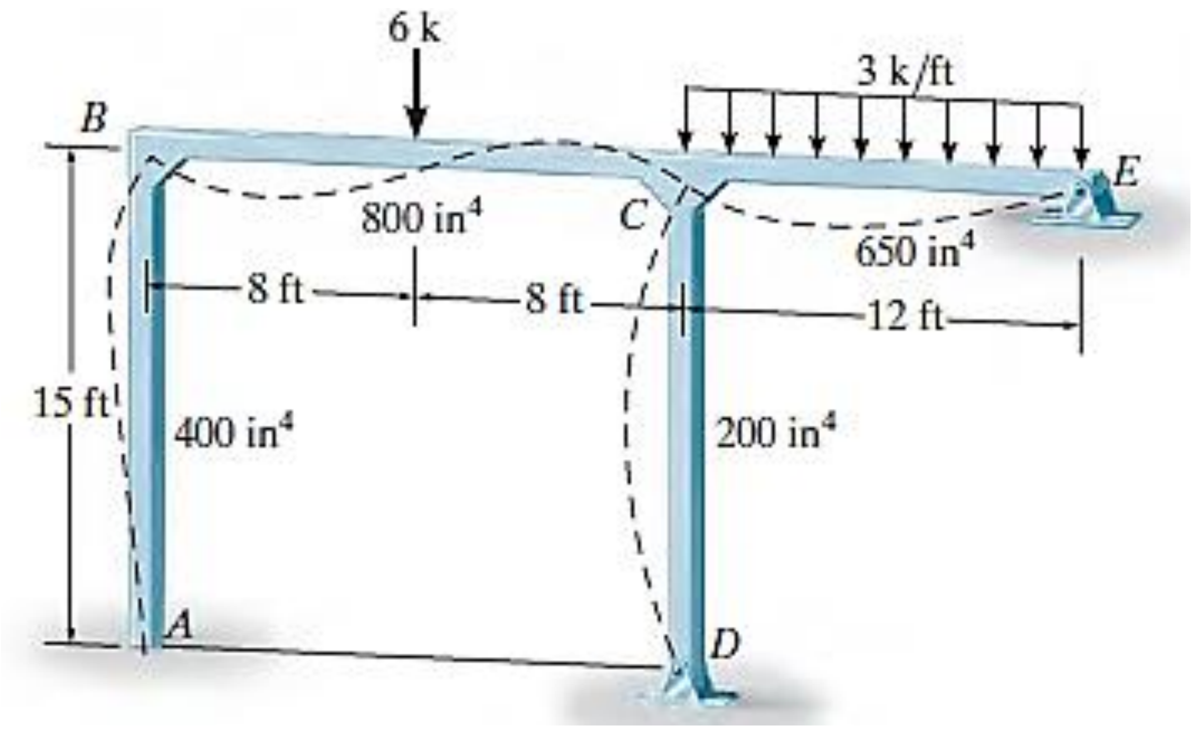
Example: Frame with no sidesway

Determine the moments at each joint of the frame. *EI* is constant.



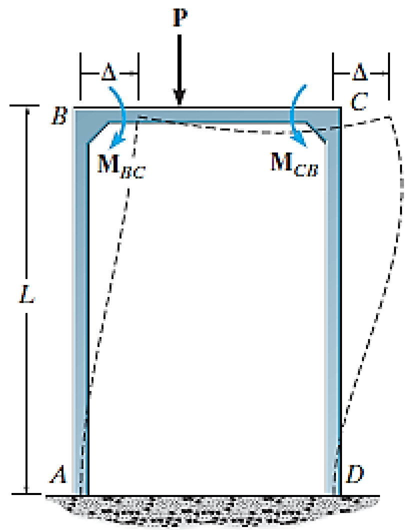
Example: Frame with no sidesway

Determine the internal moments at each joint of the frame. Take E = 2911032 ksi.



Analysis of Frames: Sidesway

- A frame will sidesway it or the loading acting on it is nonsymmetric.



- U*nequal* moments will be formed at the joints.
- When applying the slope-deflection equation to each column of this frame, the column rotation ψ must be considered as unknown in the equation.
- An extra equilibrium equation must be included for the solution.

Example: Frame with sidesway

Determine the moments at each joint of the frame. *EI* is constant.