Department of Civil Engineering

Structural analysis II

Beams and frames having nonprismatic members

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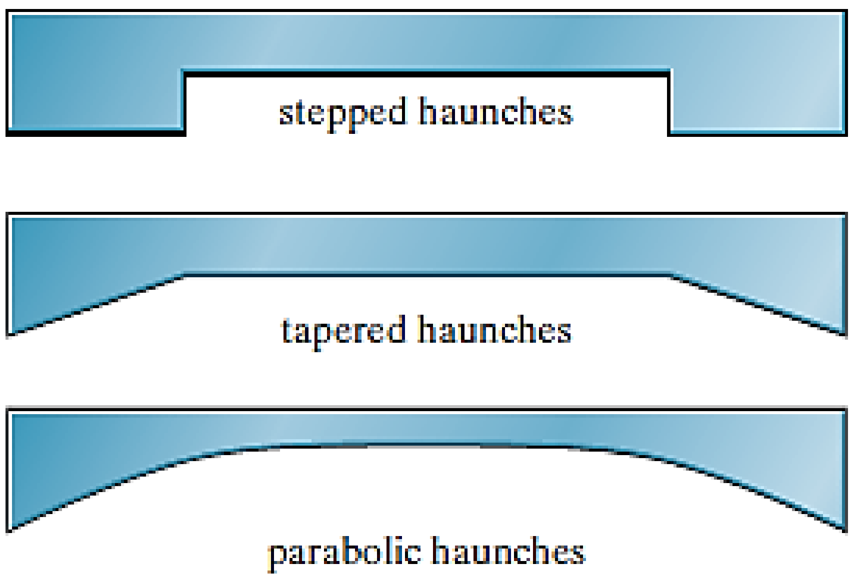
Nonprismatic members

- Often, to save material, girders used for long spans on bridges and buildings are designed to be nonprismatic, that is, to have a variable moment of inertia.
- The most common forms of structural members that are nonprismatic have haunches that are either stepped, tapered, or parabolic.



Nonprismatic members

If the slope deflection equations or moment distribution are used to determine the reactions on a nonprismatic member, then we must calculate the following properties for the member:

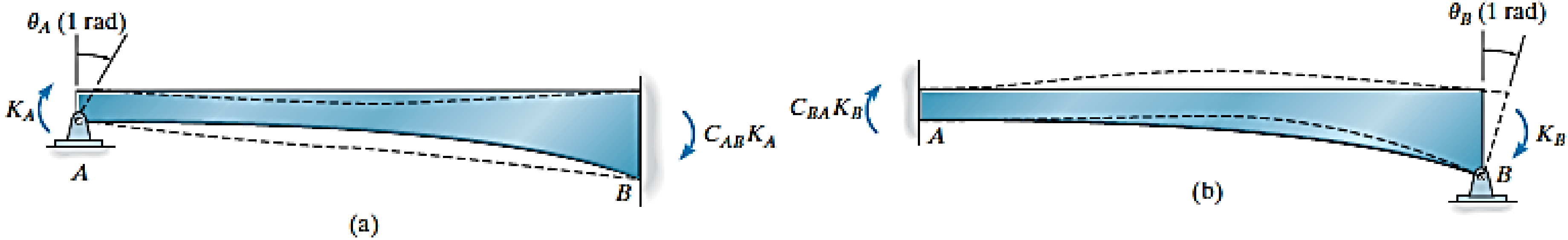
- *Fixed-End Moments (FEM).*
- *Stiffness Factor (K).*
- *Carry-Over Factor (COF).*

Once obtained, the computations for the stiffness and carry-over factors can be checked, by noting an important relationship that exists between them.

Analysis of nonprismatic members

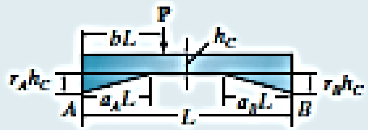
- Application of the Maxwell-Betti reciprocal theorem requires the work done by the loads in Fig. *a* acting through the displacements in Fig. *b* be equal to the work of the loads in Fig. *b* acting through the displacements in Fig. *a*, from which:

$$C_{AB}K_A = C_{BA}K_B$$



- The stiffness and carry-over factors must satisfy this condition.
- These properties can be obtained using graphs and tables have been made available to determine this data for common shapes used in structural design.

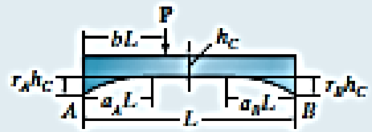
TABLE 13–1 Straight Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL										Haunch Load at			
								b										Left		Right	
								0.1		0.3		0.5		0.7		0.9		FEM Coef × $w_A L^2$		FEM Coef × $w_B L^2$	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.3$										a_B = variable				$r_A = 1.0$		r_B = variable					
	0.4	0.543	0.766	9.19	6.52	0.1194	0.0791	0.0935	0.0034	0.2185	0.0384	0.1955	0.1147	0.0889	0.1601	0.0096	0.0870	0.0133	0.0008	0.0006	0.0058
	0.6	0.576	0.758	9.53	7.24	0.1152	0.0851	0.0934	0.0038	0.2158	0.0422	0.1883	0.1250	0.0798	0.1729	0.0075	0.0898	0.0133	0.0009	0.0005	0.0060
0.2	1.0	0.622	0.748	10.06	8.37	0.1089	0.0942	0.0931	0.0042	0.2118	0.0480	0.1771	0.1411	0.0668	0.1919	0.0047	0.0935	0.0132	0.0011	0.0004	0.0062
	1.5	0.660	0.740	10.52	9.38	0.1037	0.1018	0.0927	0.0047	0.2085	0.0530	0.1678	0.1550	0.0559	0.2078	0.0028	0.0961	0.0130	0.0012	0.0002	0.0064
	2.0	0.684	0.734	10.83	10.09	0.1002	0.1069	0.0924	0.0050	0.2062	0.0565	0.1614	0.1645	0.0487	0.2185	0.0019	0.0974	0.0129	0.0013	0.0001	0.0065
	0.4	0.579	0.741	9.47	7.40	0.1175	0.0822	0.0934	0.0037	0.2164	0.0419	0.1909	0.1225	0.0856	0.1649	0.0100	0.0861	0.0133	0.0009	0.0022	0.0118
	0.6	0.629	0.726	9.98	8.64	0.1120	0.0902	0.0931	0.0042	0.2126	0.0477	0.1808	0.1379	0.0747	0.1807	0.0080	0.0888	0.0132	0.0010	0.0018	0.0124
0.3	1.0	0.705	0.705	10.85	10.85	0.1034	0.1034	0.0924	0.0052	0.2063	0.0577	0.1640	0.1640	0.0577	0.2063	0.0052	0.0924	0.0131	0.0013	0.0013	0.0131
	1.5	0.771	0.689	11.70	13.10	0.0956	0.1157	0.0917	0.0062	0.2002	0.0675	0.1483	0.1892	0.0428	0.2294	0.0033	0.0953	0.0129	0.0015	0.0008	0.0137
	2.0	0.817	0.678	12.33	14.85	0.0901	0.1246	0.0913	0.0069	0.1957	0.0750	0.1368	0.2080	0.0326	0.2455	0.0022	0.0968	0.0128	0.0017	0.0006	0.0141
$a_A = 0.2$										a_B = variable				$r_A = 1.5$		r_B = variable					
	0.4	0.569	0.714	7.97	6.35	0.1166	0.0799	0.0966	0.0019	0.2186	0.0377	0.1847	0.1183	0.0821	0.1626	0.0088	0.0873	0.0064	0.0001	0.0006	0.0058
	0.6	0.603	0.707	8.26	7.04	0.1127	0.0858	0.0965	0.0021	0.2163	0.0413	0.1778	0.1288	0.0736	0.1752	0.0068	0.0901	0.0064	0.0001	0.0005	0.0060
0.2	1.0	0.652	0.698	8.70	8.12	0.1069	0.0947	0.0963	0.0023	0.2127	0.0468	0.1675	0.1449	0.0616	0.1940	0.0043	0.0937	0.0064	0.0002	0.0004	0.0062
	1.5	0.691	0.691	9.08	9.08	0.1021	0.1021	0.0962	0.0025	0.2097	0.0515	0.1587	0.1587	0.0515	0.2097	0.0025	0.0962	0.0064	0.0002	0.0002	0.0064
	2.0	0.716	0.686	9.34	9.75	0.0990	0.1071	0.0960	0.0028	0.2077	0.0547	0.1528	0.1681	0.0449	0.2202	0.0017	0.0975	0.0064	0.0002	0.0001	0.0065
	0.4	0.607	0.692	8.21	7.21	0.1148	0.0829	0.0965	0.0021	0.2168	0.0409	0.1801	0.1263	0.0789	0.1674	0.0091	0.0866	0.0064	0.0002	0.0020	0.0118
	0.6	0.659	0.678	8.65	8.40	0.1098	0.0907	0.0964	0.0024	0.2135	0.0464	0.1706	0.1418	0.0688	0.1831	0.0072	0.0892	0.0064	0.0002	0.0017	0.0123
0.3	1.0	0.740	0.660	9.38	10.52	0.1018	0.1037	0.0961	0.0028	0.2078	0.0559	0.1550	0.1678	0.0530	0.2085	0.0047	0.0927	0.0064	0.0002	0.0012	0.0130
	1.5	0.809	0.645	10.09	12.66	0.0947	0.1156	0.0958	0.0033	0.2024	0.0651	0.1403	0.1928	0.0393	0.2311	0.0029	0.0950	0.0063	0.0003	0.0008	0.0137
	2.0	0.857	0.636	10.62	14.32	0.0897	0.1242	0.0955	0.0038	0.1985	0.0720	0.1296	0.2119	0.0299	0.2469	0.0020	0.0968	0.0063	0.0003	0.0005	0.0141

TABLE 13–2 Parabolic Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL										Haunch Load at			
								b										Left		Right	
								0.1		0.3		0.5		0.7		0.9		FEM Coef. × $w_A L^2$		FEM Coef. × $w_B L^2$	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.2$ a_B = variable $r_A = 1.0$ r_B = variable																					
	0.4	0.558	0.627	6.08	5.40	0.1022	0.0841	0.0938	0.0033	0.1891	0.0502	0.1572	0.1261	0.0715	0.1618	0.0073	0.0877	0.0032	0.0001	0.0002	0.0030
	0.6	0.582	0.624	6.21	5.80	0.0995	0.0887	0.0936	0.0036	0.1872	0.0535	0.1527	0.1339	0.0663	0.1708	0.0058	0.0902	0.0032	0.0001	0.0002	0.0031
0.2	1.0	0.619	0.619	6.41	6.41	0.0956	0.0956	0.0935	0.0038	0.1844	0.0584	0.1459	0.1459	0.0584	0.1844	0.0038	0.0935	0.0032	0.0001	0.0001	0.0032
	1.5	0.649	0.614	6.59	6.97	0.0921	0.1015	0.0933	0.0041	0.1819	0.0628	0.1399	0.1563	0.0518	0.1962	0.0025	0.0958	0.0032	0.0001	0.0001	0.0032
	2.0	0.671	0.611	6.71	7.38	0.0899	0.1056	0.0932	0.0044	0.1801	0.0660	0.1358	0.1638	0.0472	0.2042	0.0017	0.0971	0.0032	0.0001	0.0000	0.0033
	0.4	0.588	0.616	6.22	5.93	0.1002	0.0877	0.0937	0.0035	0.1873	0.0537	0.1532	0.1339	0.0678	0.1686	0.0073	0.0877	0.0032	0.0001	0.0007	0.0063
	0.6	0.625	0.609	6.41	6.58	0.0966	0.0942	0.0935	0.0039	0.1845	0.0587	0.1467	0.1455	0.0609	0.1808	0.0057	0.0902	0.0032	0.0001	0.0005	0.0065
0.3	1.0	0.683	0.598	6.73	7.68	0.0911	0.1042	0.0932	0.0044	0.1801	0.0669	0.1365	0.1643	0.0502	0.2000	0.0037	0.0936	0.0031	0.0001	0.0004	0.0068
	1.5	0.735	0.589	7.02	8.76	0.0862	0.1133	0.0929	0.0050	0.1760	0.0746	0.1272	0.1819	0.0410	0.2170	0.0023	0.0959	0.0031	0.0001	0.0003	0.0070
	2.0	0.772	0.582	7.25	9.61	0.0827	0.1198	0.0927	0.0054	0.1730	0.0805	0.1203	0.1951	0.0345	0.2293	0.0016	0.0972	0.0031	0.0001	0.0002	0.0072
$a_A = 0.5$ a_B = variable $r_A = 1.0$ r_B = variable																					
	0.4	0.488	0.807	9.85	5.97	0.1214	0.0753	0.0929	0.0034	0.2131	0.0371	0.2021	0.1061	0.0979	0.1506	0.0105	0.0863	0.0171	0.0017	0.0003	0.0030
	0.6	0.515	0.803	10.10	6.45	0.1183	0.0795	0.0928	0.0036	0.2110	0.0404	0.1969	0.1136	0.0917	0.1600	0.0083	0.0892	0.0170	0.0018	0.0002	0.0030
0.2	1.0	0.547	0.796	10.51	7.22	0.1138	0.0865	0.0926	0.0040	0.2079	0.0448	0.1890	0.1245	0.0809	0.1740	0.0056	0.0928	0.0168	0.0020	0.0001	0.0031
	1.5	0.571	0.786	10.90	7.90	0.1093	0.0922	0.0923	0.0043	0.2055	0.0485	0.1818	0.1344	0.0719	0.1862	0.0035	0.0951	0.0167	0.0021	0.0001	0.0032
	2.0	0.590	0.784	11.17	8.40	0.1063	0.0961	0.0922	0.0046	0.2041	0.0506	0.1764	0.1417	0.0661	0.1948	0.0025	0.0968	0.0166	0.0022	0.0001	0.0032
	0.4	0.554	0.753	10.42	7.66	0.1170	0.0811	0.0926	0.0040	0.2087	0.0442	0.1924	0.1205	0.0898	0.1595	0.0107	0.0853	0.0169	0.0020	0.0042	0.0145
	0.6	0.606	0.730	10.96	9.12	0.1115	0.0889	0.0922	0.0046	0.2045	0.0506	0.1820	0.1360	0.0791	0.1738	0.0086	0.0878	0.0167	0.0022	0.0036	0.0152
0.5	1.0	0.694	0.694	12.03	12.03	0.1025	0.1025	0.0915	0.0057	0.1970	0.0626	0.1639	0.1639	0.0626	0.1970	0.0057	0.0915	0.0164	0.0028	0.0028	0.0164
	1.5	0.781	0.664	13.12	15.47	0.0937	0.1163	0.0908	0.0070	0.1891	0.0759	0.1456	0.1939	0.0479	0.2187	0.0039	0.0940	0.0160	0.0034	0.0021	0.0174
	2.0	0.850	0.642	14.09	18.64	0.0870	0.1275	0.0901	0.0082	0.1825	0.0877	0.1307	0.2193	0.0376	0.2348	0.0027	0.0957	0.0157	0.0039	0.0016	0.0181

a_A, a_B = ratio of the length of haunch at ends A and B to the length of span.

b = ratio of the distance from the concentrated load to end A to the length of span.

C_{AB}, C_{BA} = carry-over factors of member AB at ends A and B, respectively.

h_A, h_B = depth of member at ends A and B, respectively.

h_C = depth of member at minimum section.

I_C = moment of inertia of section at minimum depth.

k_{AB}, k_{BA} = stiffness factor at ends A and B, respectively.

L = length of member.

M_{AB}, M_{BA} = fixed-end moment at ends A and B, respectively; specified in tables for uniform load w or concentrated force P.

r_A, r_B = ratios for rectangular cross-sectional areas, where $r_A = (h_A - h_C)/h_C$, $r_B = (h_B - h_C)/h_C$.

The fixed-end moments and carry-over factors can be found from the tables. The absolute stiffness factor then can be determined using the tabulated stiffness factors and found from:

$$K_A = \frac{k_{AB} E I_C}{L} \qquad K_B = \frac{k_{BA} E I_C}{L}$$

Moment Distribution for Structures Having Nonprismatic Members

- Once the fixed-end moments and stiffness and carry-over factors for the nonprismatic members of a structure have been determined, application of the moment-distribution method follows the same procedure as outlined in Chapter 12.
- The distribution of moments may be shortened if a member stiffness factor is modified to account for conditions of end-span pin support and structure symmetry or antisymmetry. Similar modifications can also be made to nonprismatic members.

Moment Distribution for Structures Having Nonprismatic Members

- *Beam Pin Supported at Far End.*

$$K'_A = K_A(1 - C_{AB}C_{BA})$$

- *Symmetric Beam and Loading.*

$$K'_A = K_A(1 - C_{AB})$$

- *Symmetric Beam with Antisymmetric Loading.*

$$K'_A = K_A(1 + C_{AB})$$

- *Relative Joint Translation of Beam.*

Fixed-end moment for the case of fixed far end is:

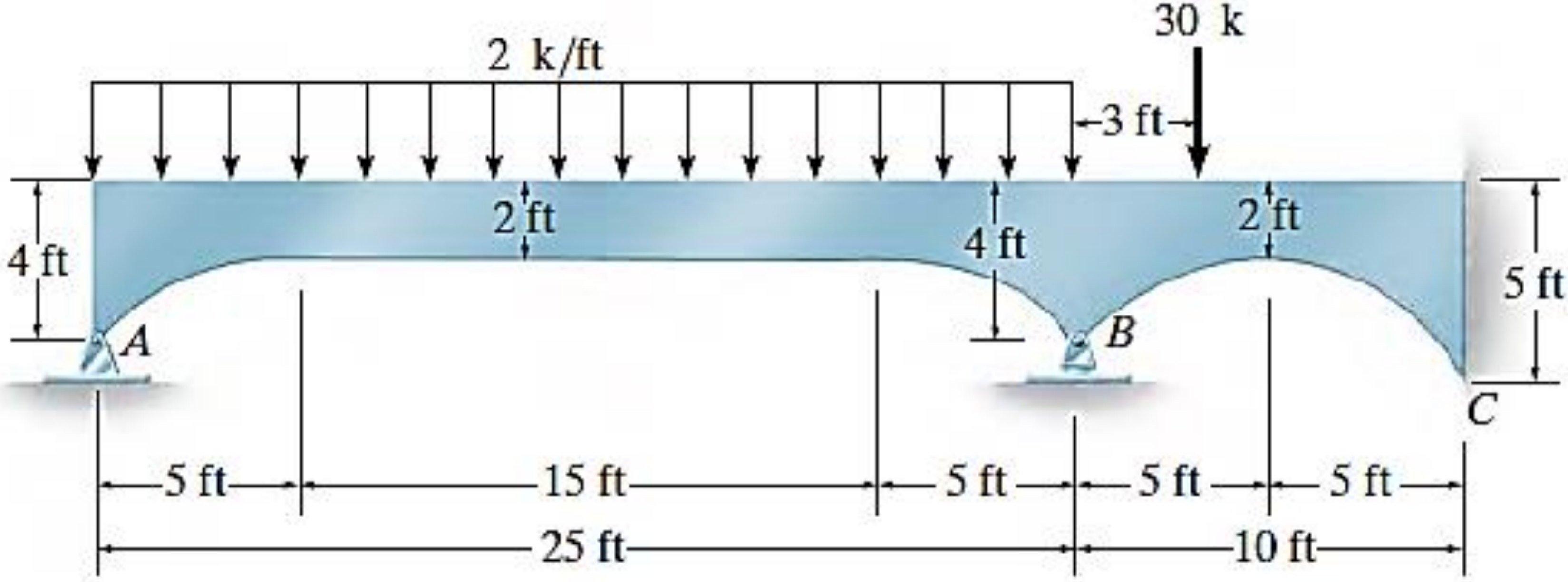
$$(\mathrm{FEM})_{AB} = -K_A\frac{\Delta}{L}(1 + C_{AB})$$

Fixed-end moment for the case of pinned far end is:

$$(\mathrm{FEM})'_{AB} = -K_A\frac{\Delta}{L}(1 - C_{AB}C_{BA})$$

Example (1): Moment distribution method for beam of nonprismatic members.

Determine the internal moments at the supports of the beam shown. The beam has a thickness of 1 ft and *E* is constant.



Solution:

Since the haunches are parabolic, we will use Table 13–2 to obtain the moment-distribution properties of the beam.

Span AB

$$a_A = a_B = \frac{5}{25} = 0.2 \qquad r_A = r_B = \frac{4-2}{2} = 1.0$$

Entering Table 13–2 with these ratios, we find

$$C_{AB} = C_{BA} = 0.619$$

$$k_{AB} = k_{BA} = 6.41$$

The stiffness of the member is:

$$K_{AB} = K_{BA} = \frac{kEI_C}{L} = \frac{6.41E\left(\frac{1}{12}\right)(1)(2)^3}{25} = 0.171E$$

Since the far end of span BA is pinned, we will modify the stiffness factor of BA:

$$K'_{BA} = K_{BA}(1 - C_{AB}C_{BA}) = 0.171E[1 - 0.619(0.619)] = 0.105E$$

For uniform load, use Table 13–2, the fixed end moments are:

$$(\text{FEM})_{AB} = -(0.0956)(2)(25)^2 = -119.50\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = 119.50\ \text{k}\cdot\text{ft}$$

Span *BC*

$$a_B = a_C = \frac{5}{10} = 0.5 \qquad r_B = \frac{4-2}{2} = 1.0$$

$$r_C = \frac{5-2}{2} = 1.5$$

From Table 13–2 we find

$$C_{BC} = 0.781 \qquad C_{CB} = 0.664$$

$$k_{BC} = 13.12 \qquad k_{CB} = 15.47$$

The stiffness of the member:

$$K_{BC} = \frac{kEI_C}{L} = \frac{13.12E\left(\frac{1}{12}\right)(1)(2)^3}{10} = 0.875E$$

$$K_{CB} = \frac{kEI_C}{L} = \frac{15.47E\left(\frac{1}{12}\right)(1)(2)^3}{10} = 1.031E$$

Concentrated load,

$$b = \frac{3}{10} = 0.3$$

$$(\text{FEM})_{BC} = -0.1891(30)(10) = -56.73 \text{ k} \cdot \text{ft}$$

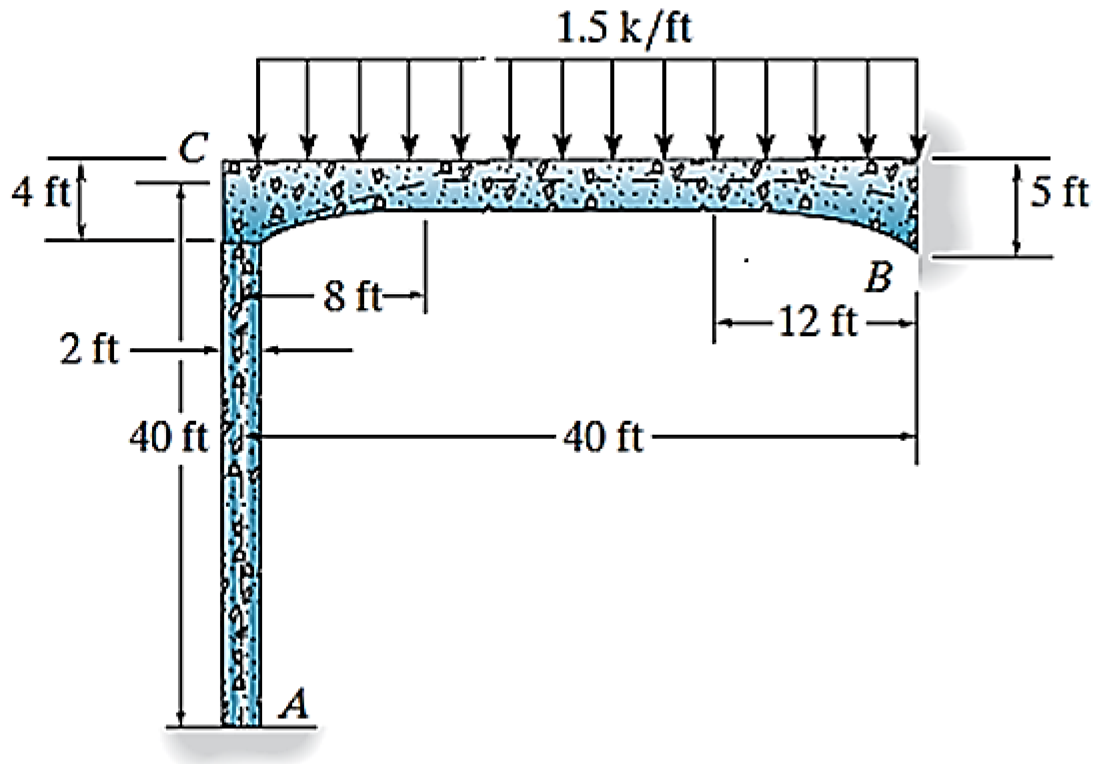
$$(\text{FEM})_{CB} = 0.0759(30)(10) = 22.77 \text{ k} \cdot \text{ft}$$

Using the foregoing values for the stiffness factors, the distribution factors are computed as summarised in the table below. The moment distribution follows the same procedure outlined in Chapter 12.

Joint	A	B		C
Member	AB	BA	BC	CB
K	$0.171E$	$0.105E$	$0.875E$	$1.031E$
DF	1	0.107	0.893	0
COF	0.619	0.619	0.781	0.664
FEM	−119.50	119.50	−56.73	22.77
Dist.	119.50	−6.72	−56.05	
CO		73.97		−43.78
Dist.		−7.91	−66.06	
CO				−51.59
ΣM	0	178.84	−178.84	−72.60

Example (2): Moment distribution method for frame of nonprismatic members.

Apply the moment-distribution method to determine the moment at each joint of the parabolic haunched frame. Supports *A* and *B* are fixed. Use Table 13–2. The members are each 1 ft thick. *E* is constant.



Slope-Deflection Equations for Nonprismatic Members

The generalized slope-deflection equation for nonprismatic members is:

$$M_N = K_N(\theta_N + C_N\theta_F - \psi(1 + C_N)) + (\text{FEM})_N$$

M_N = internal moment at the near end of the span; this moment is positive clockwise when acting on the span.

K_N = absolute stiffness of the near end determined from tables or by calculation.

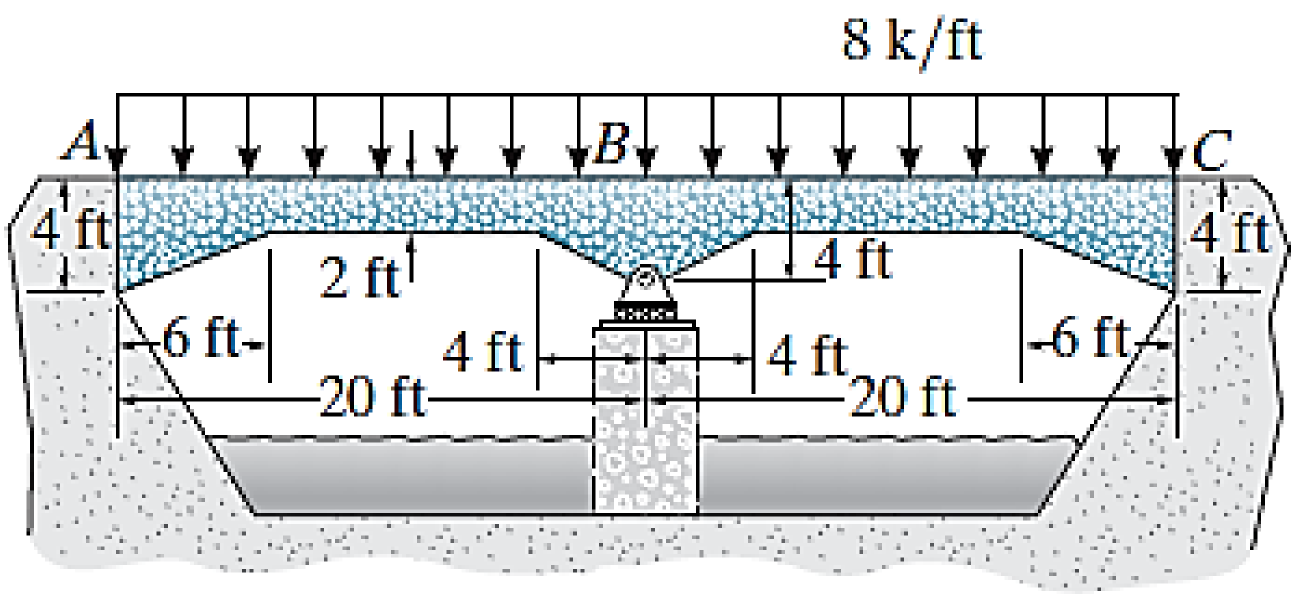
θ_N, θ_F = near- and far-end slopes of the span at the supports; the angles are measured in *radians* and are *positive clockwise.*

ψ = span cord rotation due to a linear displacement, $\psi = \Delta/L$; this angle is measured in *radians* and is *positive clockwise.*

$(\text{FEM})_N$ = fixed-end moment at the near-end support; the moment is *positive* clockwise when acting on the span and is obtained from tables or by calculations.

Example:

Determine the moments at *A*, *B*, and *C* by the the slope-deflection equations. Assume the supports at *A* and *C* are fixed and a roller support at *B* is on a rigid base. The girder has a thickness of 4 ft. *E* is constant. The haunches are straight.



$$a_A = \frac{6}{20} = 0.3 \qquad a_B = \frac{4}{20} = 0.2$$

$$r_A = r_B = \frac{4 - 2}{2} = 1$$

For span AB,

$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$$K_{BA} = \frac{K_{BA} E I_C}{L} = \frac{8.37 E I_C}{20} = 0.4185 E I_C$$

$$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44 \text{ k} \cdot \text{ft}$$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185 E I_C$

$(\text{FEM})_{BC} = -301.44 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k} \cdot \text{ft}$

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$$

$$M_{AB} = 0.503EI(0 + 0.622\theta_B - \ ) - 348.48$$

$$M_{AB} = 0.312866EI\theta_B - 348.8 \qquad (1)$$

$$M_{BA} = 0.4185EI(\theta_B + 0 - 0) + 301.44$$

$$M_{BA} = 0.4185EI\theta_B + 301.44 \qquad (2)$$

$$M_{BC} = 0.4185EI(\theta_B + 0 - 0) - 301.44$$

$$M_{BC} = 0.4185EI\theta_B - 301.44 \qquad (3)$$

$$M_{CB} = 0.503EI(0 + 0.622\theta_B - 0) + 348.48$$

$$M_{CB} = 0.312866EI\theta_B - 348.48 \qquad (4)$$

Equilibrium.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving Eqs. 1–5:

$\theta_B = 0$

$M_{AB} = -348 \text{ k} \cdot \text{ft}$

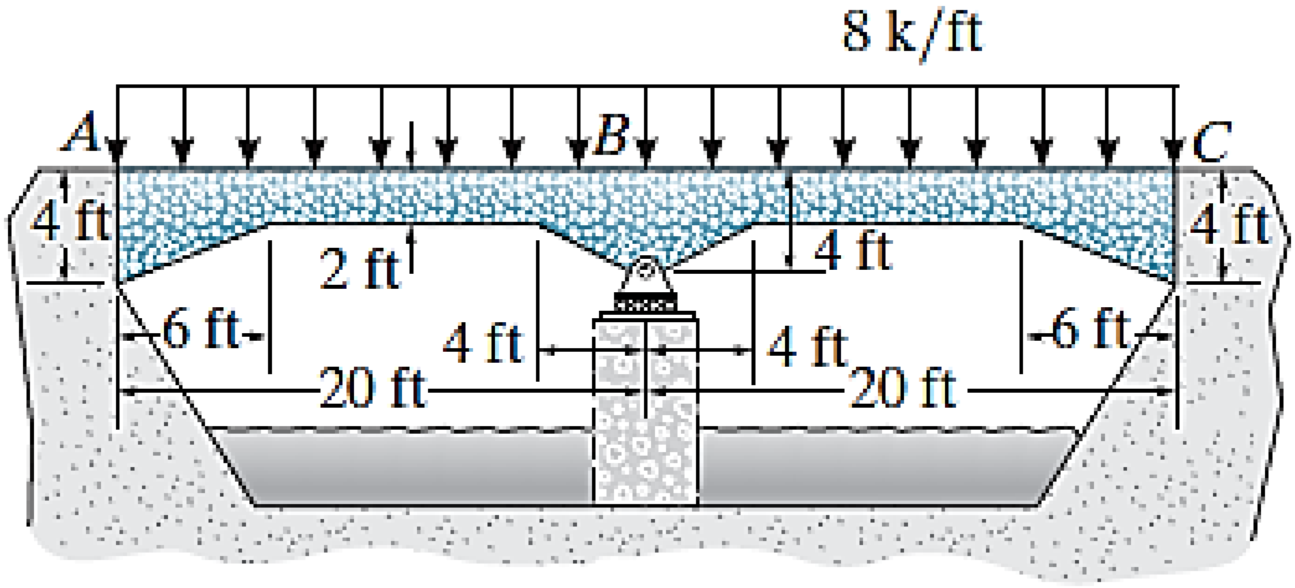
$M_{BA} = 301 \text{ k} \cdot \text{ft}$

$M_{BC} = -301 \text{ k} \cdot \text{ft}$

$M_{CB} = 348 \text{ k} \cdot \text{ft}$

Example:

Determine the moments at *A*, *B*, and *C* by the the moment distribution method. Assume the supports at *A* and *C* are fixed and a roller support at *B* is on a rigid base. The girder has a thickness of 4 ft. *E* is constant. The haunches are straight.



$$a_A = \frac{6}{20} = 0.3 \qquad a_B = \frac{4}{20} = 0.2$$

$$r_A = r_B = \frac{4-2}{2} = 1$$

For span AB,

$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$$K_{BA} = \frac{K_{BA}EI_C}{L} = \frac{8.37EI_C}{20} = 0.4185EI_C$$

$$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44 \text{ k}\cdot\text{ft}$$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185EI_C$

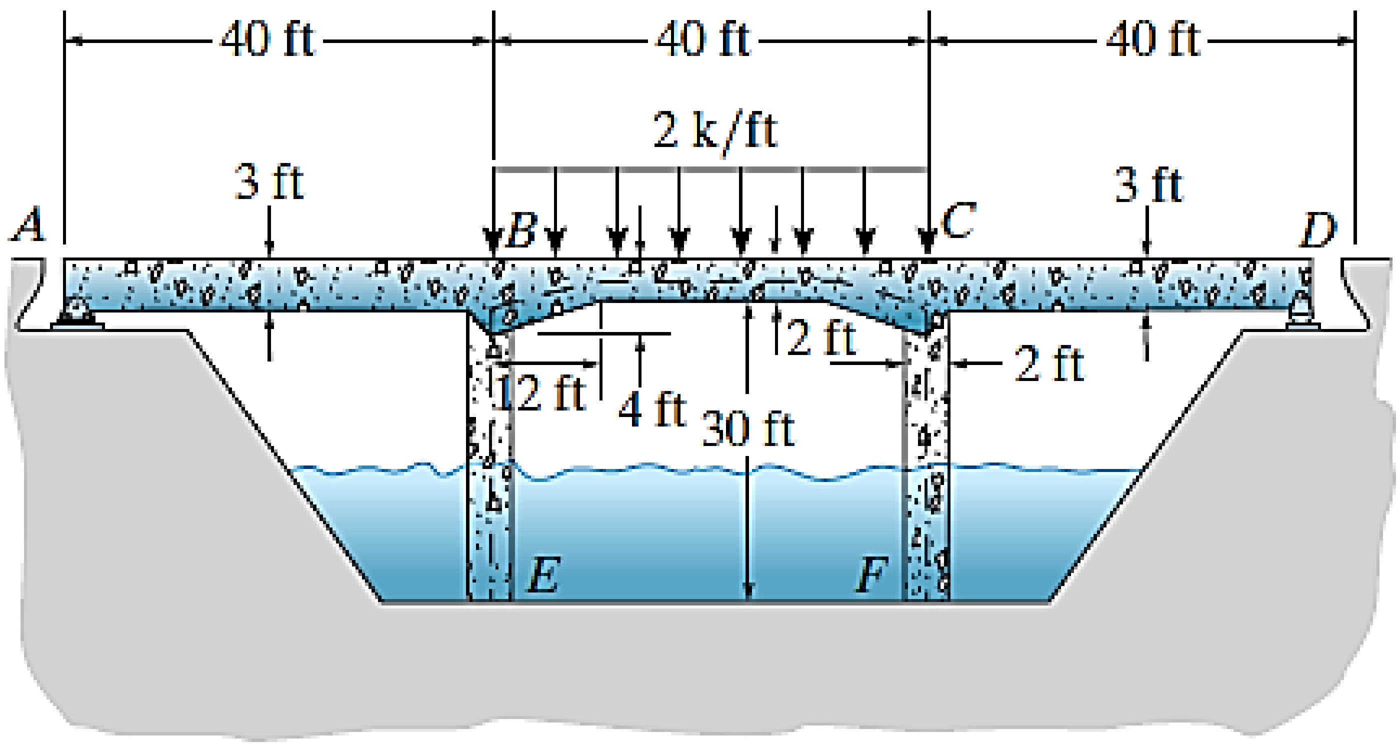
$(\text{FEM})_{BC} = -301.44 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k}\cdot\text{ft}$

Joint	A	B		C
Mem.	AB	BA	BC	CB
K		$0.4185EI_C$	$0.4185EI_C$	
DF	0	0.5	0.5	0
COF	0.622	0.748	0.748	0.622
FEM	–348.48	301.44	–301.44	348.48
		0	0	
$\sum M$	–348.48	301.44	–301.44	348.48 k · ft

Problem:

Use the moment-distribution method to determine the moment at each joint of the symmetric bridge frame. Supports *F* and *E* are fixed and *B* and *C* are fixed connected. The haunches are straight so use Table 13–2. Assume *E* is constant and the members are each 1 ft thick.



$$a_B = a_C = \frac{12}{40} = 0.3$$

$$r_B = r_C = \frac{4 - 2}{2} = 1.0$$

Thus,

$$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$$

Since the stimulate and loading are symmetry.

$$K_{BC}\frac{K_{BC}EI_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.18083E$$

$$K'_{BC} = K_{BC}(1 - C_{BC}) = 0.18083E(1 - 0.705) = 0.05335E$$

The fixed end moment are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88\ \text{k}\cdot\text{ft}$$

Since member AB and BE are prismatic

$$K_{BE} = \frac{4EI}{L_{BA}} = \frac{4E\left[\frac{1}{12}(1)(2^3)\right]}{30} = 0.08889E$$

$$K_{BA} = \frac{3EI}{L_{BA}} = \frac{3E\left[\frac{1}{12}(1)(3^3)\right]}{40} = 0.16875E$$

Joint	A	B			E
Member	AB	BA	BC	BE	EB
K		$0.16875E$	$0.05335E$	$0.08889E$	
DF	1	0.5426	0.1715	0.2859	0
COF		0	0.705	0.5	
FEM			–330.88		
Dist		179.53	56.75	94.60	
CO					47.30
$\sum M$		179.53	–274.13	94.60	47.30

Thus,

$$M_{CD} = M_{BA} = 179.53 \text{ k} \cdot \text{ft} = 180 \text{ k} \cdot \text{ft}$$

$$M_{CF} = M_{BE} = 94.60 \text{ k} \cdot \text{ft} = 94.6 \text{ k} \cdot \text{ft}$$

$$M_{CB} = M_{BC} = -274.13 \text{ k} \cdot \text{ft} = 274 \text{ k} \cdot \text{ft}$$

$$M_{FC} = M_{EB} = 47.30 \text{ k} \cdot \text{ft} = 47.3 \text{ k} \cdot \text{ft}$$

Slope-deflection method

$$a_B = a_C = \frac{12}{40} = 0.3 \qquad r_B = r_C = \frac{4-2}{2} = 1.0$$

Thus,

$$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$$

Then,

$$K_{BC} = K_{CB} = \frac{K_{BC}EI_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.1808E$$

The fixed end moment's are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88 \text{ k} \cdot \text{ft.}$$

$$\theta_C = -\theta_B$$

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(HC_N)] + (\text{FEM})_N$$

$$M_{BC} = 0.1808E[\theta_B + 0.705(-\theta_B) - 0\,(1 + 0.705)] + (-330.88)$$
$$= 0.053346E\theta_B - 330.88$$

For prismatic member BE

$$M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BE} = 2E\left[\frac{\frac{1}{12}(1)(3)^3}{30}\right][2\theta_B + 0 - 3(0)] + 0 = 0.08889E\theta_B \qquad (2)$$

$$M_{EB} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{30}\right][2(0) + \theta_B - 3(0) + 0 = 0.04444E\theta_B \qquad (3)$$

For prismatic member AB

$$M_N = 3EK(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right](\theta_B - 0) + 0 = 0.16875E\theta_B \qquad (4)$$

Moment equilibrium of joint B gives

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$0.16875E\theta_B + 0.053346E\theta_B - 330.88 + 0.08889E\theta_B = 0$$

$$\theta_B = \frac{1063.97}{E}$$

Substitute this result into Eq. (1) to (4)

$$M_{CB} = M_{BC} = -274.12 \text{ k} \cdot \text{ft} = -274 \text{ k} \cdot \text{ft}$$

$$M_{CF} = M_{BE} = 94.58 \text{ k} \cdot \text{ft} = 94.6 \text{ k} \cdot \text{ft}$$

$$M_{FC} = M_{EB} = 47.28 \text{ k} \cdot \text{ft} = 47.3 \text{ k} \cdot \text{ft}$$

$$M_{CD} = M_{BA} = 179.55 \text{ k} \cdot \text{ft} = 180 \text{ k} \cdot \text{ft}$$