Philadelphia University Faculty of Engineering Department of Computer Engineering		Date:- 29/12/2015 Allowed time:- 60 minutes

Discrete Mathematics (630260) First Exam

Student Name: - ……………………………………...................... ID: - ……………………………

Question1: Show that the following statements are logically equivalent (use equivalent rules) 6 points

((p → q) → q) → q ≡ p → q

$$((\neg p \lor q) \to q) \to q$$
$$(\neg(\neg p \lor q) \lor q) \to q$$
$$\neg(\neg(\neg p \lor q) \lor q) \lor q$$
$$((\neg p \lor q) \land \neg q) \lor q$$
$$((\neg p \land \neg q) \lor (q \land \neg q)) \lor q$$
$$(\neg p \land \neg q) \lor q$$
$$(\neg p \lor q) \land (\neg q \lor q)$$
$$(\neg p \lor q)$$
$$p \to q$$

(p ∧ q) ∨ r ≡ (p → ¬q) → r

$$(p \to \neg q) \to r$$
$$(\neg p \lor \neg q) \to r$$
$$\neg(\neg p \lor \neg q) \lor r$$
$$(p \land q) \lor r$$

(p ∨ q) ∧ (¬p → ¬q) ≡ p

$$(p \lor q) \land (p \lor \neg q)$$
$$p \land (q \lor \neg q)$$
$$p \land T$$
$$p$$

Question2: Use rules of equivalence to determine wither the following expressions are tautology or contradiction 4 points

¬(p ∨ ¬(p ∧ q))

$$\neg p \land p \land q$$
$$F \land q$$
$$F \quad \text{(contradiction)}$$

(p ∧ q) → (p ∨ q)

$$\neg(p \land q) \lor (p \lor q)$$
$$\neg p \lor q \lor p \lor q$$
$$\neg p \lor p \lor q \lor q$$
$$T \lor q$$
$$T \text{ (tautology)}$$

Question3: Use quantifiers to Construct a propositional logic expression that express the following theorem. 2 points

Let *m* be a positive integer. The integers *a* and *b* are congruent modulo *m* if and only if there is an integer *k* such that *a* = *b* + *km*.

$$\forall a \forall b \forall m((m > 0) \land \exists k(a = b + km) \leftrightarrow (a \equiv b(mod\ m)))$$

Question4: use proof by contradiction to prove that 2 points

The difference of any rational number and any irrational number is irrational.

If x is rational and y is irrational then x-y=z where z is irrational

Assume that If x is rational and y is irrational then x-y=z where z is rational

Then $\frac{a}{b} - y = \frac{c}{d}$ then $y = \frac{ad-bc}{bd}$ which is rational number but y is irrational then this is a contradiction which means If x is rational and y is irrational then x-y=z where z is irrational

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Question5: Find the solutions of the following recurrence relations. 2 points

$$a_n = 2a_{n-1} - 1\,, \quad a_0 = 1$$
$$a_1 = 2 \times 1 - 1 = 1$$
$$a_2 = 2 \times 1 - 1 = 1$$
$$a_3 = 2 \times 1 - 1 = 1$$
$$a_n = 2 \times 1 - 1 = 1$$
$$a_n = 1$$

Question6: Let A={ x,y } and B={ a,b } and C={ 1, 2} find C × A × B . 2 points

The Cartesian product of C × A × B is the set of ordered 3-tubles

C × A × B={(1,x,a),(1,x,b),(1,y,a),(1,y,b), (2,x,a),(2,x,b),(2,y,a),(2,y,b)}

Question7: Let Ai = { -∞, … , -i , i, … , ∞ } find. 2 points

1- $\bigcap_{i=0}^{n} Ai = \{-\infty, \dots\dots, -n\,, n\,, \dots\dots\dots., \infty\}$

2- $\bigcup_{i=0}^{n} Ai = \{-\infty, \infty\} = Z$

Good Luck

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