Philadelphia University Faculty of Engineering Department of Computer Engineering		Date:- 03/04/2018 Allowed time:- 60 minutes

Discrete Mathematics (630260) First Exam

Student Name: - …………………………………….................... ID: - ……………………………

Question 1:- Construct a truth table for each of the following compound proposition. **10 points**

1- $\neg p \to (q \to r)$

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

2- $(p \vee \neg t) \wedge (p \vee \neg s)$

p	t	s	$\neg t$	$\neg s$	$p \vee \neg t$	$p \vee \neg s$	$(p \vee \neg t) \wedge (p \vee \neg s)$
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	T	T	T	T
F	T	T	F	F	F	F	F
F	T	F	F	T	F	T	F
F	F	T	T	F	T	F	F
F	F	F	T	T	T	T	T

Question 2:- Show that each of the following proposition is a tautology (don't use truth table use rules of equivalence). **10 points**

1- $[\neg p \wedge (p \vee q)] \to q$

$$\neg[\neg p \wedge (p \vee q)] \vee q$$
$$[p \vee (\neg p \wedge \neg q)] \vee q$$
$$[(p \vee \neg p) \wedge (p \vee \neg q)] \vee q$$
$$[T \wedge (p \vee \neg q)] \vee q$$
$$(p \vee \neg q) \vee q$$
$$(p \vee T) \equiv T$$

2- $\neg(p \to \neg q) \to q$

$$(p \to \neg q) \vee q$$

$$(\neg p \vee \neg q) \vee q$$

$$\neg P \vee T \equiv T$$

Question 3: Show that the following pairs of propositions are equivalent. 10 points

1- $(p \to q) \wedge (p \to r)$ **and** $p \to (q \wedge r)$

$$(\neg p \vee q) \wedge (\neg p \vee r)$$

$$\neg p \vee (q \wedge r)$$

$$p \to (q \wedge r)$$

2- $\neg p \to (q \to r)$ **and** $q \to (p \vee r)$

$$p \vee (\neg q \vee r)$$

$$p \vee (\neg q \vee r)$$

$$\neg q \vee p \vee r$$

$$\neg q \vee (p \vee r)$$

$$\neg q \to (p \vee r)$$

Question 4: Let Q(x) be the statement "x + 1 > 2x." what are these truth values of the following Quantifiers with respect to given domain? 24 points

	Z (integers)	Z^+ (positive integers)	Z^- (negative integers)
$\exists x\, Q(x)$	T	F	T
$\forall x\, Q(x)$	F	F	T
$\exists x\, \neg Q(x)$	T	T	F
$\forall x\, \neg Q(x)$	F	T	F

Question 5: For each of these arguments determine whether the argument is correct or incorrect and explain why.

8 points

1- **All students in this class understand logic. sami is a student in this class. Therefore, sami understands logic.**

C(x): student x in class C
U(x): student x understand logic

$$\forall x\big(C(x) \to U(x)\big)$$
$$\underline{C(sami)}$$
$$\therefore U(sami)$$

This is a valid argument

2- **Every computer Engineering major takes discrete mathematics. Hani is taking discrete mathematics. Therefore, Hani is a computer Engineering major.**

C(x) : student x is a computer Engineer Major
D(x) : Student x takes Discrete Mathematics course

$$\forall x\big(C(x) \to D(x)\big)$$
$$\underline{D(hani)}$$
$$\therefore C(hani)$$

This is not valid argument

Question 6:- Prove that if $(7n + 4)$ is even then n is even (use contraposition principle). **6 points**

$$E(7n + 4) \rightarrow E(n)$$

Using contrapositive principle

$$\neg E(n) \rightarrow \neg E(7n + 4)$$

$$O(n) \rightarrow O(7n + 4)$$

Prove If n is odd then 7n+4 is odd

$$n = 2k + 1 \quad (odd\ number)$$

$$7n + 4 = 7(2k + 1) + 4$$

$$7n + 4 = 14k + 11 = 14k + 10 + 1 = 2(7k + 5) + 1\ which\ is\ an\ odd\ number$$

Question 7:- Let A={ 1 , 2 , 3 } and B={ a , b } and C={ 0 , 1} find C × A × B . **8 points**

$$C \times A \times B = \{(0, 1, a), (0, 1, b), (0, 2, a), (0, 2, b), (0, 3, a), (0, 3, b), (1, 1, a), (1, 1, b), (1, 2, a), (1, 2, b), (1, 3, a), (1, 3, b)\}$$

Question 8:- Let Ai = { *-∞ , … , -i-2 , -i - 1 , -i , i , i + 1 , i + 2 , … , ∞* **} find.** **8 points**

1- $\bigcap_{i=0}^{n} Ai$
2- $\bigcup_{i=0}^{n} Ai$

$$A_0 = \{-\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty\}$$
$$A_1 = \{-\infty, \dots, -2, -1, 1, 2, \dots, \infty\}$$
$$A_2 = \{-\infty, \dots, -2, , 2, \dots, \infty\}$$
$$A_n = \{-\infty, \dots, -n, , n, \dots, \infty\}$$

$$\bigcap_{i=0}^{n} Ai = \{-\infty, \dots, -n, , n, \dots, \infty\} = A_n$$
$$\bigcup_{i=0}^{n} Ai = \{-\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty\} = A_0$$

Question 9: Determine whether each of these functions is a one-to-one correspondence (bijection) from R to R.
6 points

1- $f(x) = 3x^2 + 6$
f(x) **is not one-to-one function** ***f(-n)=f(n)*** **and not onto function (negative number in codomain do not have preimage in the domain) so** ***f(x)*** **is not one-to-one correspondence (not bijection)**

2- $f(x) = 2x + 1$
f(x) **is one-to-one function (linear function) and onto function (negative number in codomain have preimage in the domain) so** ***f(x)*** **is one-to-one correspondence (bijection)**

Question 10: find the solution of the following recurrences **10 points**

1- $a_n = (n+1)a_{n-1}$ $a_0 = 2$

$$a_0 = 2$$
$$a_1 = 2 \times 2$$
$$a_2 = 3 \times 2 \times 2$$
$$a_3 = 4 \times 3 \times 2 \times 2$$

$$a_n = (n+1) \times n \times (n-1) \times \ldots \times 4 \times 3 \times 2 \times 2 = 2(n+1)!$$

2- $a_n = -2a_{n-1}$ $a_0 = 1$

$$a_0 = 1$$
$$a_1 = -2 \times 1 = -2$$
$$a_2 = -2 \times -2 = 4$$
$$a_3 = -2 \times 4 = -8$$
$$a_4 = -2 \times -8 = 16$$

$$a_n = (-2)^n$$