


Philadelphia University Faculty of Engineering Department of Computer Engineering		Second Semester 2017/2018 Date:- 29/05/2018 Allowed time:- 2 Hours
Discrete Mathematics (630260)		Final Exam
Student Name: -		ID: -

Question 1: **6 points**

A). State the converse, contrapositive, and inverse of the following statements

If n is odd then $1-n$ is even

B). Show that $\neg(p \oplus q)$ and $(p \leftrightarrow q)$ are logically equivalent.

Question 2: Prove that if n is an integer and $n^3 + 5$ is odd, then n is even using: **6 points**

- 1- a proof by contrapositive.
- 2- a proof by contradiction.

Question 3: **8 points**

A). Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

- a) $A \cup B$. b) $A \cap B$. c) $A - B$. d) $B - A$.

B). What can you say about the sets A and B if we know that

- a) $A \cup B = A$ b) $A \cap B = A$
c) $A - B = A$ d) $A - B = B - A$

Question 4: **6 points**

A). Determine whether each of these functions is a bijection from \mathbb{R} to \mathbb{R} .

1. $f(x) = -3x + 4$ 2. $f(x) = -3x^2 + 7$ 3. $f(x) = \frac{x+1}{x+2}$ 4. $f(x) = 2x^3 - 5$

Question 5: **15 points**

A). Use Chinese remainder theorem to find the solution of the following system of linear congruence's

$$\begin{aligned} x &\equiv 1 \pmod{2} \\ x &\equiv 2 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 4 \pmod{11} \end{aligned}$$

Question 6: The following message was encrypted using RSA algorithm with key $(n=119, e=77)$ **20 points**

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- 1- Determine the block size.
- 2- Find the primes factorization
- 3- Find the decryption key
- 4- Restore the original message.

Question 7: **10 points**

A). Use mathematical induction to prove the followings

1. $3^n < n!$ for $n > 6$
2. $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! = (n+1)! - 1$

B). Give a recursive definition for $a_n = n^2$ then design a recursive algorithm to find n^2

Question 8: R_1 and R_2 are relations defined on set $A = \{1,2,3,4\}$ where

$$R_1 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (3,3), (4,2), (4,3), (4,4)\}$$

$$R_2 = \{(1,1), (1,3), (1,4), (2,3), (2,4), (3,3), (3,4), (4,1), (4,2), (4,3)\}$$

10 points

A). Represent these relations using Zero-One Matrix then determine whether these relations are reflexive, symmetric and/or antisymmetric.

B). Find $1 \cup R_2$, $R_1 \cap R_2$ and $R_1 \circ R_2$ using Zero-One Matrices.

Question 9: R_3, R_4 and R_5 are relations defined on set $A = \{0,1,2,3\}$ where

$$R_3 = \{(0,0), (0,2), (1,1), (2,0), (2,2), (2,3), (3,2), (3,3)\}$$

$$R_4 = \{(0,0), (1,1), (2,2), (3,3), (1,2), (1,3), (3,2)\}$$

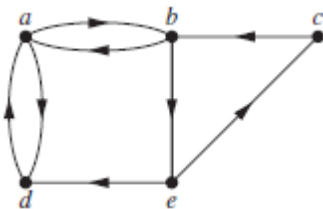
$$R_5 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,2), (3,3)\}$$

Represent the relations above using directed graphs, then determine whether the relations are equivalence or partially ordered. **10 points**

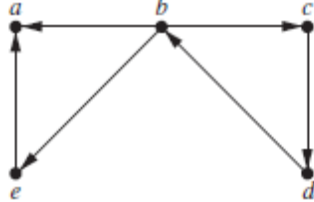
Question 10:

9 points

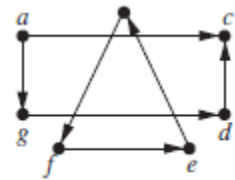
A). Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.



G1

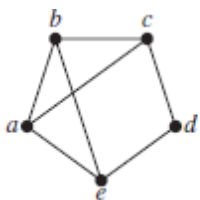


G2

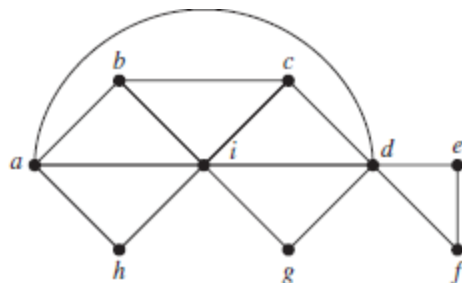


G3

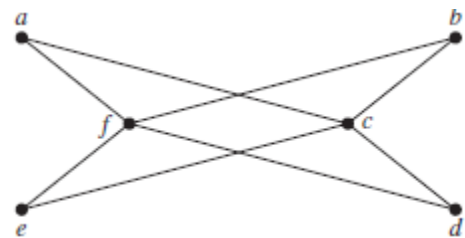
B). Determine whether each of the following graphs is bipartite or not then determine whether each one has an Euler circuit, Euler path or not



G4



G5



G6