



Discrete Mathematics (630260) Second Exam

Student Name: - ..... ID: - .....

**Question 1:** If  $a \equiv 7 \pmod{15}$  and  $b \equiv 9 \pmod{15}$  then find the followings  
10 points

$$c \equiv 3a^2 - 6b \pmod{15}$$

**Question 2:** Solve the following congruence:  
10 points  
 $23x \equiv 21 \pmod{27}$

**Question 3:** The following message was encrypted using affine cipher function  $C = (7P + 3) \pmod{26}$  where P is the original character and C cipher character decrypt to message.  
25 points

**GSNF**

**Question 4:** Use mathematical induction to prove the following summation  
10 points

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$$

**Question 5:** 20 points

- 1- Give a recursive definition for  $a_n = n(n + 1)$
- 2- Using the recursive definition of part 1 write a recursive algorithm to compute

$$\sum_{i=1}^n i(i + 1)$$

**Question 6:** Determine whether the following relation on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive.  
5 points

$$R = \{(a, b) \mid a \equiv b \pmod{7}\}$$

**Question 7:** Given that  $A = \{1, 2, 3\}$  is a set and the following relations are on A.  
20 points

$$R1 = \{(1,1), (1,3), (2,2), (3,1)\}$$

$$R2 = \{(1,2), (2,2), (3,2)\}$$

- 1- Represent these relation using Zero-One matrix then determine wither these relations reflexive, symmetric and/or antisymmetric.

- 2- Use Zero-One matrices defined above to find  $R1 \cup R2$  and  $R2 \circ R1$