Philadelphia University
Faculty of Engineering

Student Name:
Student Number

Dept. of Computer Engineering
Final Exam, Second Semester: 2017/2018

Course Title: Engineering Analysis II (630262)	**Date: 3/6/2018**	**Time Allowed: 2 hours**

NOTES: - Round ALL your calculations to 4 significant digits
- Angles for trigonometric functions are in radian scale

Question number	Q1 / 15	Q2 / 16	Q3 / 15	Q4 / 10	Q5 / 20	Q6 / 24	Total / 100
Grade							

Please choose your instructor:

Instructor:	☐ **Dr. Mohammed Mahdi**	☐ **Eng. Anis Nazer**	☐ **Eng. Sultan Al-Rushdan**
Lecture time:	☐ **9:10 ح ث خ**	☐ **11:10 ح ث خ**	☐ **13:10 ح ث خ** ☐ **9:45 ن ر**

Question 1: (15 marks)

Perform three false position iterations (find x_1 , x_2 , x_3) to approximate the solution to the following equation, the root is between [2 , 4]

$$\sin(x)=\ln(x)-1.5$$

x_L	x_U	$f(x_L)$	$f(x_U)$	x_M	$f(x_M)$
2	4	1.7162	-0.6431	3.4548	-0.04791
2	3.4548	1.7162	-0.0479	3.4153	1.4123E-3
3.4153	3.4548	1.4123E-3	-0.0479	3.4164	

Question 2: **(16 marks)**

You are required to use Gauss-Seidel iterations to solve the following system of linear equations:

$$\begin{bmatrix} 3.1 & -4.9 & 1.1 \\ 2.5 & -1.3 & 4.9 \\ -5.7 & 1.2 & -2.0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 3 \end{bmatrix}$$

a) Rearrange the system to make sure that the iterations will converge

b) Start with $x=0, \quad y=0, \quad z=0$ and perform 2 Gauss-Seidel iterations

$$\begin{bmatrix} -5.7 & 1.2 & -2.0 \\ 3.1 & -4.9 & 1.1 \\ 2.5 & -1.3 & 4.9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ 3 \end{bmatrix} \quad \rightarrow \quad \begin{aligned} x &= \frac{3-(1.2y-2z)}{-5.7} \\ y &= \frac{12-(3.1x+1.1z)}{-4.9} \\ z &= \frac{3-(2.5x-1.3y)}{4.9} \end{aligned}$$

x	y	z
0	0	0
-0.5263	-2.7820	0.1427
-1.1621	-3.1521	0.3689

Another rearrangement:

$$\begin{bmatrix} -4.9 & 1.1 & 3.1 \\ -1.3 & 4.9 & 2.5 \\ 1.2 & -2 & -5.7 \end{bmatrix} \begin{bmatrix} y \\ z \\ x \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 3 \end{bmatrix} \quad \rightarrow \quad \begin{aligned} y &= \frac{12-(1.1z+3.1x)}{-4.9} \\ z &= \frac{3-(-1.3y+2.5x)}{4.9} \\ x &= \frac{3-(1.2y-2z)}{-5.7} \end{aligned}$$

y	z	x
0	0	0
-2.4490	-0.0375	-1.0287
-3.1082	0.3125	-1.2903

Question 3: **(15 marks)**

The distance of a car is recorded at different times. Use 2nd order Lagrange interpolation to find the relation between the time and distance. **Write the resulting function in the form** $f(x)=a_0+a_1 x+a_2 x^2$

x : time is seconds	5	10	15
f(x) : distance in meter/second	342	872	1530

Assuming $x_0=5$, $x_1=10$, $x_2=15$

$$L_0(x)=\left(\frac{x-10}{5-10}\right)\left(\frac{x-15}{5-15}\right)=\frac{1}{50}(x^2-25x+150)$$

$$L_1(x)=\left(\frac{x-5}{10-5}\right)\left(\frac{x-15}{10-15}\right)=\frac{1}{-25}(x^2-20x+75)$$

$$L_2(x)=\left(\frac{x-5}{15-5}\right)\left(\frac{x-10}{15-10}\right)=\frac{1}{50}(x^2-15x+50)$$

$$f(x)=\frac{1}{50}(x^2-25x+150)(342)+\frac{1}{-25}(x^2-20x+75)(872)+\frac{1}{50}(x^2-15x+50)(1530)$$

$$f(x)=\left(\frac{342}{50}-\frac{872}{25}+\frac{1530}{50}\right)x^2+\left(\frac{-25\times 342}{50}+\frac{-20\times 872}{-25}+\frac{-15\times 1530}{50}\right)x+\left(\frac{150\times 342}{50}+\frac{75\times 872}{-25}+\frac{50\times 1530}{50}\right)$$

$$f(x)=2.56x^2+67.6x-60$$

Question 4: (10 marks)

Given the following data, use linear regression to find the relation between x and y, then find SSE for that relation

x	y
1.0	-0.76
1.5	0.55
2.0	1.83
2.5	3.00

x^2	$x\,y$	$f(x)$	$error$	$error^2$
1	-0.76	-0.729	0.031	0.000961
2.25	0.825	0.527	0.023	0.000529
4	3.66	1.783	0.047	0.002209
6.25	7.5	3.039	0.039	0.001521

$$\sum x=7 \quad , \quad \sum y=4.62 \quad , \quad \sum x^2=13.5 \quad , \quad \sum x\,y=11.225$$

13.5 A + 7 B = 11.225
7 A + 4 B = 4.62

Solving the equations:

$$A=\frac{11.225\times 4-7\times 4.62}{13.5\times 4-7\times 7}=2.512$$

$$B=\frac{13.5\times 4.62-11.225\times 7}{13.5\times 4-7\times 7}=-3.241$$

→ $f(x)=2.512\,x-3.241$

$$\sum error^2=0.000961+0.000529+0.002209+0.001521=0.00522$$

Question 5: **(20 marks)**

Consider the following differential equation:

$$y' = -2y + 5e^{-t} \qquad , \qquad y(2) = 1.5$$

a) Approximate y at x = 2.6, using Euler method with a **step size of 0.2**
b) Approximate y at x = 2.6, using Heun's method with a **step size of 0.3**
c) Which approximation is better, given that the true value is $y(2.6)=0.619348$

a) Euler method:

t	y	f(t,y)
2	1.5	-2.3233
2.2	1.0353	-1.5167
2.4	0.7320	-1.0104
2.6	0.5299	

b) Heun's method:

t	y	K1	t+h	y+hK1	K2
2	1.5	-2.3233	2.3	0.8030	-1.1047
2.3	0.9858	-1.4703	2.6	0.5447	-0.7180
2.6	0.6575				

c)
Error in Euler : $E_{abs}=0.0894$, $E_{rel}=14.4\%$
Error in Heun : $E_{abs}=0.0382$, $E_{rel}=6.17\%$

→ Heun's method gives a better approximation than Euler

Question 6: **(24 marks)**

Choose the correct answer in the following questions (3 marks each)

Question	Answer
1) If the computer stores $\sqrt{5}$ as the number 2.2360680103, then the approximation is true for _____ significant digits? Assume that the true value of $\sqrt{5}$ is 2.236067977 A) 6 **B) 7** C) 8 D) cannot be determined	B
2) When using interpolation with 4 points, the order of the resulting polynomial **may be** __?___ A) first order B) 2nd order C) 3rd order **D) All of the choices**	D
3) Which of the following formulas can be used to solve $xe^x=1$ using Newton Raphson iterations **A)** $x=\frac{x^2e^x+1}{e^x(x+1)}$ B) $x=\frac{x^2e^x+1}{e^x}$ C) $x=\frac{xe^x-1}{e^x(x+1)}$ D) None of the choices	A
4) Which of the following is true? ($[A],[B],[C]$ are square matrices) A) $[A][B]=[B][A]$ **B)** $[A][I]=[I][A]$ C) $([A]^T)^T=[A]^{-1}$ D) $[A]([B][C])=([A][C])[B]$	B
5) The eigen values of $\begin{bmatrix} a_{11} & 0 \\ -1 & 4 \end{bmatrix}$ are $\lambda_1=1$ and $\lambda_2=4$, then a_{11} = A) 0 B) 2.1324 **C) 1** D) 4	C
6) Given the integral $\int_1^2 e^{-x}dx$, which of the following will result in the best approximation? (the true value is 0.2325441579) A) using Composite Trapezoidal with h=0.2 **B) using Composite 1/3 Simpson with h=0.1** C) using Composite 1/3 Simpson with 7 points D) Cannot be detirmined	B
7) A square matrix [A] is lower triangular if: A) $a_{ij}=0$ for all i = j B) $a_{ij}=0$ for all i > j **C)** $a_{ij}=0$ **for all i < j** D) $a_{ij}\neq 0$ for all i > j	C
8) The polynomial that passes through the points (18,24) (22,25) (24,123) is given by $y=8.125x^2-324.75x+3237$ The function using Newton's divided difference polynomial is given by $f_2(x)=b_0+b_1(x-18)+b_2(x-18)(x-22)$ The value of b_2 is : **A) 8.125** B) 24 C) 3237 D) 0.25	A

GOOD LUCK