Modeling and Simulation
Revision IV

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Modeling

- **Modeling is the process of representing the behavior of a real system** by a collection of mathematical equations and logic.

- Models are cause-and-effect structures—they accept external information and process it with their logic and equations to produce one or more outputs.
  - Parameter is a fixed-value unit of information
  - Signal is a changing-unit of information

- Models can be text-based programming or block diagrams
Math Modelling Categories

- Static vs. dynamic
- Linear vs. nonlinear
- Time-invariant vs. time-variant
- SISO vs. MIMO
- Continuous vs. discrete
- Deterministic vs. stochastic
Static vs. Dynamic

- Models can be static or dynamic
  - **Static models** produce no motion, fluid flow, or any other changes.
    - Example: Battery connected to resistor $v = iR$
  - **Dynamic models** have energy transfer which results in power flow. This causes motion, or other phenomena that change in time.
    - Example: Battery connected to resistor, inductor, and capacitor
      \[
      v = Ri + L \frac{di}{dt} + \int \frac{1}{C} idt
      \]
Linear vs. Nonlinear

- **Linear** models follow the **superposition principle**
  - The summation outputs from individual inputs will be equal to the output of the combined inputs

A system represented by $S$ is said to be *linear* if for inputs $x(t)$ and $v(t)$, and any constants $\alpha$ and $\beta$, superposition holds—that is,

$$
S[\alpha x(t) + \beta v(t)] = S[\alpha x(t)] + S[\beta v(t)] \\
= \alpha S[x(t)] + \beta S[v(t)]
$$

- Most systems are nonlinear in nature, but linear models can be used to approximate the nonlinear models at certain point.
Linear vs. Nonlinear Models

- **Linear Systems**

\[ \ddot{x}(t) = -\frac{B}{M} \dot{x}(t) - \frac{K}{M} (x(t) - x_0) + \frac{1}{M} F(t) \]

- **Nonlinear Systems**

\[ -F_T \cos \theta + mg = m \left( -\ell \ddot{\theta} \sin \theta - \ell \dot{\theta}^2 \sin \theta \right) \]

\[ -F_T \sin \theta = m \left( \ell \ddot{\theta} \cos \theta - \ell \dot{\theta}^2 \sin \theta \right) \]
Time-invariant vs. Time-variant

- The model parameters do not change in time-invariant models
- The model parameters change in time-variant models
  - Example: Mass in rockets vary with time as the fuel is consumed.

*If the system parameters change with time, the system is time varying.*
## Time-invariant vs. variant

<table>
<thead>
<tr>
<th>Time-invariant</th>
<th>Time-variant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(t) = ma(t) - g$</td>
<td>$F = m(t) a(t) - g$</td>
</tr>
</tbody>
</table>

*Where $m$ is the mass, $a$ is the acceleration, and $g$ is the gravity*

*Here, the mass varies with time. Therefore the model is time-varying*
**Linear Time-Invariant (LTI)**

- **LTI** models are of great use in representing systems in many engineering applications.
  - The appeal is its simplicity and mathematical structure.

- Although most actual systems are nonlinear and time varying
  - Linear models are used to approximate around an operating point the nonlinear behavior
  - Time-invariant models are used to approximate in short segments the system’s time-varying behavior.
SISO vs. MIMO

- **Single-Input Single-Output** (SISO) models are somewhat easy to use. Transfer functions can be used to relate input to output.

- **Multiple-Input Multiple-Output** (MIMO) models involve combinations of inputs and outputs and are difficult to represent using transfer functions. *MIMO models use State-Space equations*
System States

- **Transfer functions**
  - Concentrates on the input-output relationship only.
  - Relates output-input to one-output only **SISO**
  - It hides the details of the inner workings.

- **State-Space Models**
  - *States* are introduced to get better insight into the systems’ behavior. These states are a collection of variables that summarize the present and past of a system.
  - Models can be used for **MIMO** models
SISO vs. MIMO Systems

Transfer Function

U(t) → Transfer Function → Y(t)
Continuous vs. discrete

- **Continuous models** have continuous-time as the dependent variable and therefore inputs-outputs take all possible values in a range.

- **Discrete models** have discrete-time as the dependent variable and therefore inputs-outputs take on values at specified times only in a range.
Continuous vs. discrete

- Continuous Models
  - Differential equations
  - Integration
  - Laplace transforms

- Discrete Models
  - Difference equations
  - Summation
  - Z-transforms
Deterministic vs. Stochastic

- **Deterministic models** are uniquely described by mathematical equations. Therefore, all past, present, and future values of the outputs are known precisely.

- **Stochastic models** cannot be described mathematically with a high degree of accuracy. These models are based on the theory of probability.
Block Diagrams

- Block diagram models consist of two fundamental objects: *signal blocks and wires*.
  - A *block* is a processing element which operates on input signals and parameters to produce output signals.
  - A wire is to transmit a signal from its origination point (usually a block) to its termination point (usually another block).

- Block diagrams are suitable to represent multi-disciplinary models that represent a physical phenomenon.

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Block Diagram Example

THREE BLOCK SYSTEM EXAMPLE

[Ref.] Prof. Shetty
Block Diagrams Manipulation

SERIES MANIPULATION—SERIES BLOCKS MULTIPLY

PARALLEL MANIPULATION—PARALLEL BLOCKS ADD

BASIC FEEDBACK SYSTEM (BFS) BLOCK DIAGRAM

\[ \frac{Y}{R} = \frac{G(D)}{1 + G(D) \cdot H(D)} \]
Block Diagrams Manipulation

**PICK-OFF POINT SHIFTED DOWNSTREAM**

\[
\begin{align*}
\text{Left: } & \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \quad \Rightarrow \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \\
& \quad z \rightarrow C \\
\text{Right: } & \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \\
& \quad z \rightarrow C \rightarrow 1/B 
\end{align*}
\]

**PICK-OFF POINT SHIFTED UPSTREAM**

\[
\begin{align*}
\text{Left: } & \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \quad \Rightarrow \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \\
& \quad z \rightarrow C \\
\text{Right: } & \quad x \rightarrow A \rightarrow w \rightarrow B \rightarrow y \\
& \quad z \rightarrow C \rightarrow B 
\end{align*}
\]
Block Diagrams: Direct Method Example

Consider the transfer function:

\[
T(s) = \frac{Y(s)}{R(s)} = \frac{s^2 - 3s + 4}{s^4 + 2s^3 - 5s^2 + 2s - 9}
\]

We can introduce a state variable, \( x(t) \), in order to separate the polynomials.

![Block Diagram for the transfer function]
The differential equation is:

\[
\frac{d^4x(t)}{dt^4} + 2 \frac{d^3x(t)}{dt^3} - 5 \frac{d^2x(t)}{dt^2} + 2 \frac{dx(t)}{dt} - 9x(t) = r(t)
\]

Put the needed \textit{integrator blocks}:

Add the required \textit{multipliers} to obtain the state equation:
Repeat the same procedure for the output equation:

\[ \ddot{x}(t) - 3\dot{x}(t) + 4x(t) = y(t) \]

Connect the two sub-blocks
Physical laws are used to predict the behavior (both static and dynamic) of systems.
- Electrical engineering relies on Ohm’s and Kirchoff’s laws
- Mechanical engineering on Newton’s law
- Electromagnetics on Faradays and Lenz’s laws
- Fluids on continuity and Bernoulli’s law

Based on electrical analogies, we can derive the fundamental equations of systems in five disciplines of engineering:
- Electrical, Mechanical, Electromagnetic, Fluid, and Thermal.

By using this analogy method to first derive the fundamental relationships in a system, the equations then can be represented in block diagram form, allowing secondary and nonlinear effects to be added.
- This two-step approach is especially useful when modeling large coupled systems using block diagrams.
### TABLE 9.1 Power and Energy Variables for Mechanical Systems

<table>
<thead>
<tr>
<th>Energy Domain</th>
<th>Effort, $e$</th>
<th>Flow, $f$</th>
<th>Power, $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>$e$</td>
<td>$f$</td>
<td>$e \cdot f$ [W]</td>
</tr>
<tr>
<td>Translational</td>
<td>Force, $F$ [N]</td>
<td>Velocity, $V$ [m/sec]</td>
<td>$F \cdot V$ [N m/sec, W]</td>
</tr>
<tr>
<td>Rotational</td>
<td>Torque, $T$ or $\tau$ [N m]</td>
<td>Angular velocity, $\omega$ [rad/sec]</td>
<td>$T \cdot \omega$ [N m/sec, W]</td>
</tr>
<tr>
<td>Electrical</td>
<td>Voltage, $v$ [V]</td>
<td>Current, $i$ [A]</td>
<td>$v \cdot i$ [W]</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure, $P$ [Pa]</td>
<td>Volumetric flowrate, $Q$ [m$^3$/sec]</td>
<td>$P \cdot Q$ [W]</td>
</tr>
</tbody>
</table>

[Ref.] Raul Longoria
Transitional Mechanical Systems

- Mechanical movements in a straight line (i.e. linear motion) are called “transitional”
- Basic Blocks are: Dampers, Masses, and Springs
- Springs represent the stiffness of the system
- Dampers (or dashpots) represent the forces opposing to the motion (i.e. friction)
- Masses represent the inertia

\[ F = kx \]

\[ F = cv \]

\[ F = ma \]
Transitional Mechanical Systems

- Equations for mechanical systems are based on Newton Laws
- Free body diagram

\[ ma = F - kx - c \frac{dx}{dt} \]
Example: Mass-Spring-Damper

\[ \ddot{x}(t) = -\frac{B}{M} \dot{x}(t) - \frac{K}{M} (x(t) - x_0) + \frac{1}{M} F(t) \]

Note:  
D is Differentiation  
1/D is Integration
Example: Two-Mass Mechanical System

Mass 1: \[ \ddot{x}_1(t) = \frac{1}{M_1} \sum F_1(t) \]

Mass 2: \[ \ddot{x}_2(t) = \frac{1}{M_2} \sum F_2(t) \]
Example: Two-Mass Mechanical System

\[ \sum F_1(t) = F_1(t) - K_1(x_1(t) - x_2(t)) - B(\dot{x}_1(t) - \dot{x}_2(t)) \]

\[ \sum F_2(t) = K_1(x_1(t) - x_2(t)) + B(\dot{x}_1(t) - \dot{x}_2(t)) - K_2 x_2(t) \]
Example: Mechanical Model

- Consider a two carriage train system

\[ m_1 \ddot{x}_1 = f - k(x_1 - x_2) - c\dot{x}_1 \]
\[ m_2 \ddot{x}_2 = k(x_1 - x_2) - c\dot{x}_2 \]
Example continued

- Taking the Laplace transform of the equations gives

\[ m_1 s^2 X_1(s) = F(s) - k(X_1(s) - X_2(s)) - csX_1(s) \]
\[ m_2 s^2 X_2(s) = k(X_1(s) - X_2(s)) - csX_2(s) \]

- Note: Laplace transforms the time domain problem into s-domain (i.e. frequency)

\[ L\{x(t)\} = X(s) = \int_0^\infty e^{-st} x(t)dt \]
\[ L\{\dot{x}(t)\} = sX(s) \]
Manipulating the previous two equations, gives the following transfer function (with F as input and V1 as output)

\[
\frac{V_1(s)}{F(s)} = \frac{m_2s^2 + cs + k}{m_1m_2s^3 + c(m_1 + m_2)s^2 + (km_1 + km_2 + c^2)s + 2kc}
\]

Note: Transfer function is a frequency domain equation that gives the relationship between a specific input to a specific output.
• Simulation using MATLAB

- \( m_1 = 5; \ m_2 = 0.7; \ k = 0.8; \ c = 0.05; \)
- \( \text{num} = [m_2 \ c \ k]; \)
- \( \text{den} = [m_1 \cdot m_2 \ c \cdot m_1 + c \cdot m_2 \ k \cdot m_1 + k \cdot m_2 + c \cdot c \ 2 \cdot k \cdot c]; \)
- \( \text{sys} = \text{tf(num,den); } \% \text{constructs the transfer function} \)
- \( \text{impulse(sys); } \% \text{plots the impulse response} \)
- \( \text{step(sys); } \% \text{plots the step response} \)
- \( \text{bode(sys); } \% \text{plots the Bode plot} \)
Example continued: Impulse response
Example continued: Step response
Example continued: Bode Plot

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Example: Motion of Aircraft

(a) Harrier “jump jet”

(b) Simplified model

\((x, y, \theta)\) denote the position and orientation of the center of mass

\[
\begin{align*}
    m\ddot{x} &= F_1 \cos \theta - F_2 \sin \theta - cx, \\
    m\ddot{y} &= F_1 \sin \theta + F_2 \cos \theta - mg - cy, \\
    J\ddot{\theta} &= rF_1.
\end{align*}
\]
Consider a mechanical system that involves rotation.

- The torque, T, replaces the force, F.
- The angle, \( \theta \), replaces the displacement \( x \).
- The angular velocity, \( \omega \), replaces velocity \( v \).
- The angular acceleration, \( \alpha \), replaces the acceleration \( a \).
- The moment of inertia \( J \), replaces the mass \( m \).

\[
\omega = \frac{d\theta}{dt}
\]

\[
J \frac{d\omega}{dt} + k\theta + c\omega
\]
Rotational Mechanical Systems

- The mechanics equation becomes

\[ T = k\theta + c\omega + J\alpha \]

\[ \Rightarrow T = k\theta + c\frac{d\theta}{dt} + J\frac{d^2\theta}{dt^2} \]
Example: Rotational-Transitional System

- Consider a rack-and-pinion system. The rotational motion of the pinion is transformed into transitional motion of the rack.

\[ T_{in} - T_{out} = J \frac{d\omega}{dt} + c_1\omega \]

For simplicity, the spring effects are ignored.

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The rotational equation is

\[ T_{in} - T_{out} = J \frac{d\omega}{dt} + c_1 \omega \]

The transitional equation is

\[ F - c_2 v = m \frac{dv}{dt} \]

Using the equations

\[ T_{out} = rF \]
\[ \omega = \frac{v}{r} \]

And manipulating the rotational and transitional equations with the input torque, \( T_{in} \), as inputs and velocity, \( v \), as output, we get

\[ T_{in} = \left( \frac{c_1}{r} + c_2 r \right) v + \left( \frac{J}{r} + mr \right) \frac{dv}{dt} \]
Example continued

Let us take a look at the state space equations

In general,

\[ \dot{x} = Ax + Cu \]
\[ y = Bx + Du \]

where \( x \) is the states vector, \( y \) is the output vector, and \( u \) is the input vector

In our example, we will use the states: \( \omega \) and \( v \), the inputs: \( T_{in} \) and \( F \), the output: \( v \)

Manipulating the equations in the previous slide, we get

\[
\begin{bmatrix}
\frac{d\omega}{dt} \\
\frac{dv}{dt}
\end{bmatrix} =
\begin{bmatrix}
-c_1/J & 0 \\
0 & -c_2/m
\end{bmatrix}
\begin{bmatrix}
\omega \\
v
\end{bmatrix} +
\begin{bmatrix}
1/J & -r/J \\
0 & 1/m
\end{bmatrix}
\begin{bmatrix}
T_{in} \\
F
\end{bmatrix}
\]

\[
v = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\
v \end{bmatrix}
\]
Conversion: Transitional and Rotational

\[ J = \frac{W}{g} \left( \frac{L}{2\pi} \right)^2 \]

\[ J = Mr^2 = \frac{W}{g} r^2 \]

\[ J = Mr^2 = \frac{W}{g} r^2 \]
Gear Trains

\[
\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}
\]

Inertia: \( \left( \frac{N_1}{N_2} \right)^2 J_2 \)

Viscous-friction coefficient: \( \left( \frac{N_1}{N_2} \right)^2 B_2 \)

Torque: \( \frac{N_1}{N_2} T_2 \)

Angular displacement: \( \frac{N_1}{N_2} \theta_2 \)

Angular velocity: \( \frac{N_1}{N_2} \omega_2 \)

Coulomb friction torque: \( \frac{N_1}{N_2} F_{c2} \omega_2 / |\omega_2| \)
Gear Trains

\[ T(t) = J_1 \frac{d^2 \theta_1(t)}{dt^2} + B_1 \frac{d \theta_1(t)}{dt} + F_{c1} \frac{\omega_1}{|\omega_1|} + T_1(t) \]

\[ T_2(t) = J_2 \frac{d^2 \theta_2(t)}{dt^2} + B_2 \frac{d \theta_2(t)}{dt} + F_{c2} \frac{\omega_2}{|\omega_2|} \]

\[ T_1(t) = \frac{N_1}{N_2} T_2(t) = \left( \frac{N_1}{N_2} \right)^2 J_2 \frac{d^2 \theta_1(t)}{dt^2} + \left( \frac{N_1}{N_2} \right)^2 B_2 \frac{d \theta_1(t)}{dt} + \frac{N_1}{N_2} F_{c2} \frac{\omega_2}{|\omega_2|} \]

\[ T(t) = J_{1e} \frac{d^2 \theta_1(t)}{dt^2} + B_{1e} \frac{d \theta_1(t)}{dt} + T_F \]

where

\[ J_{1e} = J_1 + \left( \frac{N_1}{N_2} \right)^2 J_2 \]

\[ B_{1e} = B_1 + \left( \frac{N_1}{N_2} \right)^2 B_2 \]

\[ T_F = F_{c1} \frac{\omega_1}{|\omega_1|} + \frac{N_1}{N_2} F_{c2} \frac{\omega_2}{|\omega_2|} \]
Electrical Systems: Basic Equations

- **Resistor**
  - Ohm’s Law

- **Inductor**

- **Capacitor**

**Power** = Voltage \( \times \) Current

- **Voltage** \( V = Ri \)**
- **Inductance** \( V = L \frac{di}{dt} \)**
- **Capacitance** \( V = \int \frac{1}{C} i \, dt \)

\( \Rightarrow i = C \frac{dV}{dt} \)**

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Kirchoff Laws

Equations for electrical systems are based on Kirchoff’s Laws

1. **Kirchoff current law:**
   Sum of Input currents at node = Sum of output currents

2. **Kirchoff voltage law:**
   Summation of voltage in closed loop equals zero
Example: RLC circuit

Using Kirchoff voltage law

\[ V = Ri + L \frac{di}{dt} + \int \frac{1}{C} i dt \quad \text{Or} \quad V = Ri + L \frac{di}{dt} + V_c \]

since \( i = C \frac{dV_c}{dt} \) Then

\[ V = RC \frac{dV_c}{dt} + LC \frac{d^2 V_c}{dt^2} + V_c \]

A second order differential equation
RLC MATLAB Code

- \( R = 1000000; \quad \% \ R = 1 \text{M}\Omega \)
- \( L = 0.001; \quad \% \ L = 1 \text{ mH} \)
- \( C = 0.000001; \quad \% \ C = 1 \mu\text{F} \)
- \( \text{num}=1; \quad \text{den}=[L*C \ R*C \ 1]; \)
- \( \text{sys} = \text{tf} (\text{num}, \text{den}); \)
- \( \text{bode} (\text{sys}) \)
- \( \text{Impulse} (\text{sys}) \)
- \( \text{Step} (\text{sys}) \)
RLC Simulation: Bode Plot

At DC (i.e. frequency = 0),
Capacitor is open
=> Voltage gain is 0 dB (i.e. 1 V/V)

At high frequency,
Capacitor is short
=> Voltage = 0
RLC Simulation: Impulse Response

Input voltage is pulse
implies Capacitor stores energy

And then releases the energy
RLC Simulation: Step Response

At about 2.3 seconds, the capacitor Voltage becomes 90% of the 1 Volts

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Op Amps

\[ G(s) = \frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \]

<table>
<thead>
<tr>
<th>Input Element</th>
<th>Feedback Element</th>
<th>Transfer Function</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 [ Z_1 = R_1 ]</td>
<td>0 [ Z_2 = R_2 ]</td>
<td>[ \frac{R_2}{R_1} ]</td>
<td>Inverting gain, e.g., if ( R_1 = R_2 ), ( e_o = -e_1 )</td>
</tr>
<tr>
<td>0 [ Z_1 = R_1 ]</td>
<td>0 [ C_2 ]</td>
<td>( \left( \frac{-1}{R_1 C_2} \right) \frac{1}{s} )</td>
<td>Pole at the origin, i.e., an integrator</td>
</tr>
<tr>
<td>0 [ Y_1 = s C_1 ]</td>
<td>0 [ Z_2 = R_2 ]</td>
<td>( (-R_2 C_1) s )</td>
<td>Zero at the origin, i.e., a differentiator</td>
</tr>
</tbody>
</table>
The electrical equation is
\[ V_{in} = Ri + L \frac{di}{dt} + V_{emf} \]
where \( V_{emf} \) (Back electromagnetic voltage) = \( k_1 \omega \)

The mechanical equation is
\[ T = J \frac{d\omega}{dt} + bw + T_{load} \]
where \( T = k_2 i \)
DC Motor Model: Block Diagram

\[ V_{in} \xrightarrow{} + \xrightarrow{} 1/(Ls+R) \xrightarrow{1} i \xrightarrow{k_2} T \xrightarrow{-} 1/(Js+b) \xrightarrow{+} \omega \]

\[ T = T_{load} \]

\[ k_1 \]
Simulation Result
Fluid Systems

- Fluid systems can be divided into two categories:
  - Hydraulic: fluid is a liquid and incompressible
  - Pneumatic: fluid is gas and can be compressed

- The volumetric rate of flow, \( q \), is equivalent to the current
- The pressure difference, \( P_1 - P_2 \), is equivalent to voltage

- The basic building blocks for hydraulic systems are:
  Hydraulic resistance, capacitance, and inertance
Hydraulic resistance

- Hydraulic resistance is the resistance to the fluid flow which occurs as a result of valves or pipe diameter changes.

- The relationship between the volume rate of flow, $q$, and pressure difference, $p_1 - p_2$, is given by Ohm’s law.

$$p_1 - p_2 = Rq$$
Hydraulic Capacitance

- Potential energy stored in a liquid such as height of a liquid in a container

\[ \frac{\partial V}{\partial t} = V = Ah \Rightarrow q_1 - q_2 = A \frac{dh}{dt} \]

\[ q_1 - q_2 = \frac{dV}{dt} \]
Hydraulic Capacitance

\[ p_1 - p_2 = p = h \rho g \]

pressure \hspace{1cm} height \hspace{1cm} gravity

Note that \( p = F / A = mg / A \Rightarrow p = \rho V g / A \Rightarrow p = h \rho g \)

\[ \frac{q_1 - q_2}{dt} = A \frac{dh}{dt} \Rightarrow q_1 - q_2 = A \left( \frac{d(p/\rho g)}{dt} \right) = A \frac{dp}{g \rho \ dt} \]

By letting the hydraulic capacitance be \( C = \frac{A}{g \rho} \)

We get \( q_1 - q_2 = C \frac{dp}{dt} \)
Hydraulic Inertance

- Equivalent to inductance in electrical systems
- To accelerate a fluid and increase its velocity a force is required

\[ F_1 - F_2 = (p_1 - p_2)A \]

Using

\[ F_1 - F_2 = ma \Rightarrow m \frac{dv}{dt} \]

\[ m = AL\rho \]

\[ q = Av \]

Then

\[ p_1 - p_2 = l \frac{dq}{dt} \]

Where the Inertance is

\[ I = \frac{L\rho}{A} \]
Hydraulic Example Modeling: an interactive 2-tank system

\[ \frac{dh_1}{dt} = \frac{q_{in}(t) - q_1(t)}{A_1} \]

\[ \frac{dh_2}{dt} = \frac{q_1(t) - q_2(t)}{A_2} \]

\[ q_1(t) = \frac{h_1(t) - h_2(t)}{R_1} \]

\[ q_2(t) = \frac{h_2(t)}{R_2} \]
Hydraulic Example Modeling: Block Diagram

Input: $q_{in}$

Output: $q_2$

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Hydraulic Example: Simulation

Input, $q_{in}$, is a step

Output, $q_2$, is taken to a virtual scope

Here, we assume all the Cross sectional areas and the resistances equals 1
Another Form of Analogies
Potential and Flow Variables

- When systems are in motion, the energy can be
  - Increased by an energy-producing source outside the system
  - Distributed between components within the system
  - Decreased by energy loss through components out of the system.

- Therefore, a coupled system becomes synonymous with energy transfer between systems.

\[
\text{Potential Variable} = PV \\
\text{Flow Variable} = FV
\]
## Analogies: FV and PV

<table>
<thead>
<tr>
<th></th>
<th>Flow Variable (FV)</th>
<th>Potential Variable (PV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electrical</strong></td>
<td>Current</td>
<td>Voltage</td>
</tr>
<tr>
<td><strong>Mechanical</strong></td>
<td>Force</td>
<td>Velocity</td>
</tr>
<tr>
<td>Transitional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotational</td>
<td>Torque</td>
<td>Angular Velocity</td>
</tr>
<tr>
<td><strong>Hydraulic</strong></td>
<td>Volumetric Flow Rate</td>
<td>Pressure</td>
</tr>
<tr>
<td><strong>Pneumatic</strong></td>
<td>Mass Flow Rate</td>
<td>Pressure</td>
</tr>
<tr>
<td><strong>Thermal</strong></td>
<td>Heat Flow Rate</td>
<td>Temperature</td>
</tr>
</tbody>
</table>
Which Analogies to use?

- **Force-Voltage** makes more physical sense
  - Graphical Representation: Bond Graphs
- **Force-Current** makes mathematical sense
- **Sum of Currents** = Zero and **Sum of Forces** = Zero
  - Graphical Representation: Linear Graphs
Conclusion

- Mathematical Modeling of physical systems is an essential step in the design process.

- Simulation should follow the modeling in order to investigate the system response.

- Mechatronic systems involve different disciplines and therefore an appropriate modeling technique to use is block diagrams.

- Analogies among disciplines can be used to simplify the understanding of different dynamic behaviors.

Dr. Tarek A. Tutunji